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Name.............................

Reg. No...........................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2008

EN 04 101—MATHEMATICS—I

(2004 Admissions)

Time : Three Hours | Maximum : 100 Marks

Part A

I. (a) Find the centre of curvature at $\left(\frac{a}{4}, \frac{a}{4}\right)$ on $\sqrt{x} + \sqrt{y} = \sqrt{a}$.

(b) Show that the function $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$ has a maximum at $(-7, -7)$ and a minimum at $(3, 3)$.

(c) Discuss the convergence for the series

$$\frac{x}{1+x} + \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} + \ldots + \frac{x^n}{1+x^n} + \ldots.$$

(d) Expand $x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$ upto the third degree terms.

(e) Verify Cayley-Hamilton theorem and find A^{-1} for $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$.

(f) Find the rank of the matrix by reducing it to the normal form :

$$\begin{pmatrix} 1 & -2 & 3 & 1 \\ 2 & 3 & 1 & 1 \\ 4 & -1 & 7 & 3 \\ 5 & 4 & 5 & 3 \end{pmatrix}.$$

(g) Obtain the Fourier series to represent x^2 from $x = -l$ to $x = l$.

(h) If $f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$ and

$f(x + 2\pi) = f(x)$ for all x. Derive the Fourier series for $f(x)$.

(8 × 5 = 40 marks)

Turn over

Part B

II. (a) (i) For the curve $\frac{\sqrt{x}}{\sqrt{a}} + \frac{\sqrt{y}}{\sqrt{b}} = 1$, show that the radius of curvature is $\frac{2(ax+by)^{3/2}}{ab}$.

(8 marks)

(ii) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. (7 marks)

Or

(b) (i) Find the minimum value of $x^2 + y^2 + z^2$ where $ax + by + cz = p$. (8 marks)

(ii) Find the maximum and minimum of $x^m y^n z^p$ such that $ax + by + cz = p + q + r$.

(7 marks)

III. (a) (i) Show that the series $\sum \frac{\{(n+1)\,r\}^n}{n^{r+1}}$ is convergent if $r < 1$ and divergent if $r \geq 1$.

(8 marks)

(ii) Expand $x^2y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$ upto third degree terms. (7 marks)

Or

(b) (i) Test the convergence of the series $\left(\frac{2}{3}\right)x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \ldots$ (8 marks)

(ii) Expand $21 + x - 20y + 4x^2 + xy + 6y^2$ in powers of $(x+1)$ and $(y-2)$. (7 marks)

IV. (a) (i) Solve the system of equations by Gauss-elimination method :

$$\begin{aligned} x_1 + x_2 + 2x_3 - x_4 &= 5 \\ x_1 + x_2 + 3x_3 + 2x_4 &= 20 \\ x_1 + 3x_2 + 2x_3 + x_4 &= 17 \\ x_1 + 3x_2 + 4x_3 + 2x_4 &= 27 \end{aligned}$$

(8 marks)

(ii) Find the value of a and b for which the equations $x + y + 2z = 2$; $2x - y + 3z = 2$; $5x - y + az = b$ have : (1) No solution ; (2) a unique solution ; (3) an infinite number of solution.

(7 marks)

Or

(b) Reduce $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form by an orthogonal reduction. Also find rank and nature of the quadratic form.

(15 marks)

V. (a) (i) Find the Fourier series of $f(x) = x + x^2$ from $(-\pi, \pi)$ and deduce that :

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots.$$

(8 marks)

(ii) Obtain the half-range sine series of the function $f(x) = kx(x - l)$ in $0 \le x \le l$. (7 marks)

Or

(b) (i) Obtain the constant term and the coefficients for the first sine and cosine terms in the Fourier series represents y as given in the following table :

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(8 marks)

(ii) Find the half-range Forier cosine series for $f(x) = \pi^2 - x^2$ in $(0, \pi)$. (7 marks)

[4 × 15 = 60 marks]