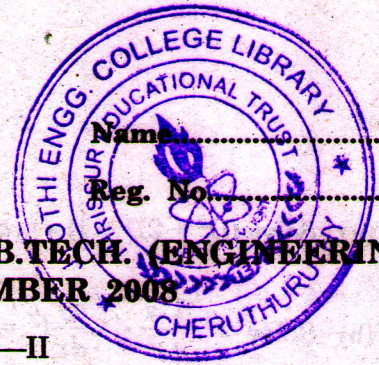


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**COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING)
DEGREE EXAMINATION, DECEMBER 2008**

EN 2K 102—MATHEMATICS—II

(Common to all Branches)

Time : Three Hours

Maximum : 100 Marks

I. (a) Solve $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$.

(b) Solve $(D^2 - 4)y = x^2$.

(c) Find the Laplace transform $\frac{1 - e^t}{t}$.

(d) Solve the integral equation $y + \int_0^t y dt = 1 - e^{-t}$.

(e) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.

(f) Prove that $\text{div curl } \vec{F} = 0$.

(g) Verify Green's theorem in the plane for $\oint_C (xy + y^2) dx + x^2 dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

(h) Evaluate $\iiint_V \nabla \cdot \vec{F} dv$. Where $\vec{F} = 2x^2y \vec{i} - y^2 \vec{j} + 4xz^2 \vec{k}$ and V is the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and $x = 2$.

(8 × 5 = 40 marks)

Turn over

II. (a) Solve $(D^2 + 2D + 1)y = e^{-x} + 3$.

(7 marks)

(b) Solve $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = \log x$.

(8 marks)

Or

(b) (i) Find the orthogonal trajectories of the family of cardioids $r = Q(1 - \cos \theta)$ where " Q " is a parameter.

(7 marks)

(ii) Solve by method of variation of parameters $\frac{d^2 y}{dx^2} + 4y = \tan 2x$.

(8 marks)

III. (a) (i) Find the Laplace transform of the function :

$$f(t) = \begin{cases} \sin wt & 0 < t < \pi/w \\ 0 & \pi/w = t < 2\pi/w \end{cases}$$

(7 marks)

(ii) Find the inverse transform of :

$$1 \quad \frac{1}{s(s^2 + a^2)}$$

$$2 \quad \frac{1}{s(s+1)^3}$$

(8 marks)

Or

(b) (i) Find $L \left\{ \frac{\cos 2t - \cos 3t}{t} \right\}$.

(7 marks)

(ii) Use Laplace transform, solve $\frac{d^2 y}{dt^2} + \frac{dy}{dt} = t^2 + 2t$, given that $y = 4, \frac{dy}{dt} = -2$ when $t = 0$.

(8 marks)

IV. (a) (i) Find the scalar potential ϕ if $\nabla\phi = 2xyz \vec{i} + x^3z \vec{j} + x^2y \vec{k}$. (7 marks)

(ii) Find the equation of the tangent plane and normal line to the surface $xyz = 4$ at the point

$$\vec{i} + 2\vec{j} + 2\vec{k}.$$

(8 marks)

Or

(b) (i) Prove that $\nabla \times (\phi \vec{F}) = (\nabla\phi) \times \vec{F} + \phi (\nabla \times \vec{F})$. (7 marks)

(ii) Find the value of λ if the vector $(\lambda x^2y + yz) \vec{i} + (xy^2 - xz^2) \vec{j} + (2xyz - 2x^2y^2) \vec{k}$ has

zero divergence. Also find the curl of the above vector when it has zero divergence.

(8 marks)

V. (a) Evaluate $\iint_S \vec{F} \cdot \vec{n} ds$ where $\vec{F} = 18z \vec{i} - 12 \vec{j} + 3y \vec{k}$ as the part of the plane $2x + 3y + 6z = 12$

which is in the first octant.

(15 marks)

Or

(b) Verify the divergence theorem, for $\vec{F} = 4x \vec{i} - 2y \vec{j} + z^2 \vec{k}$ taken over the region bounded by

$$x^2 + y^2 = 4, z = 0 \text{ and } z = 3.$$

(15 marks)

[4 × 15 = 60 marks]