D 51210 **(Pages 3)**

Name..

Reg. No....................................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2008

EN 2K 102—MATHEMATICS—II

(Common to all Branches)

Time : Three Hours Maximum : 100 Marks

I. (a) Solve $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$.

(b) Solve $(D^2 - 4)\, y = x^2$.

(c) Find the Laplace transform $\frac{1 - e^t}{t}$.

(d) Solve the integral equation $y + \int_0^t y\,dt = 1 - e^{-t}$.

(e) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.

(f) Prove that div curl $\vec{F} = 0$.

(g) Verify Greetis theorem in the plane for $\oint \left(xy + y^2\right) dx + x^2\, dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

(h) Evaluate $\iiint_v \nabla \cdot \vec{F}\, dv$. Where $\vec{F} = 2x^2 y\, \vec{i} - y^2\, \vec{j} + 4xz^2\, \vec{k}$ and V is the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and $x = 2$.

(8 × 5 = 40 marks)

Turn over

II. (a) Solve $(D^2 + 2D + 1)\, y = e^{-x} + 3.$ (7 marks)

(b) Solve $x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} t^y = \log x.$ (8 marks)

Or

(b) (i) Find the orthogonal tranectories of t he family of cardioids $\gamma = Q\,(1 - \cos\theta)$ where "a" is a parameters.

(7 marks)

(ii) Solve by method of variation of parameters $\dfrac{d^2y}{dx^2} + 4y = \tan 2x.$ (8 marks)

III. (a) (i) Find the Laplace transform of the function :

$$f(t) = \begin{cases} \sin wt & 0 < t < \pi/w \\ 0 & \pi/w = t < 2\pi/w \end{cases}.$$

(7 marks)

(ii) Find the inverse transform of :

1 $\dfrac{1}{s\left(s^2 + a^2\right)}$

2 $\dfrac{1}{s(s+1)^3}.$

(8 marks)

Or

(b) (i) Find $L\left\{\dfrac{\cos 2t - \cos 3t}{t}\right\}.$ (7 marks)

(ii) Use Laplace transform, solve $\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} = t^2 + 2t,$ given that $y = 4,\ \overset{0}{y} = -2$ when $t = 0.$

(8 marks)

IV. (a) (i) Find the scalar potential ϕ if $\nabla\phi = 2xyz\,\vec{i} + x^3z\,\vec{j} + x^2y\,\vec{k}$. (7 marks)

(ii) Find the equation of the tangent plane and normal line to the surface $xyz = 4$ at the point $\vec{i} + 2\vec{j} + 2\vec{k}$.

(8 marks)

Or

(b) (i) Prove that $\nabla \times \left(\phi\vec{F}\right) = (\nabla\phi)\times\vec{F}\,\phi\left(\nabla\times\vec{F}\right)$. (7 marks)

(ii) Find the value of λ if the vector $\left(\lambda x^2y + yz\right)\vec{i} + \left(xy^2 - xz^2\right)\vec{j} + \left(2xyz - 2x^2y^2\right)\vec{k}$ has zero divergence. Also find the curl of the above vector when it has zero divergence.

(8 marks)

V. (a) Evaluate $\iint\limits_{S} \vec{F}\cdot\vec{n}\,ds$ where $\vec{F} = 18z\,\vec{i} - 12\,\vec{j} + 3y\,\vec{k}$ as the part of the plane $2x + 3y + 6z = 12$ which is in the first octant.

(15 marks)

Or

(b) Verify the divergence theorem, for $\vec{F} = 4x\,\vec{i} - 2y\,\vec{j} + z^2\,\vec{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.

(15 marks)

[4 × 15 = 60 marks)]