D 51209 **(Pages 3)**

Name..............................

Reg. No..............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2008

EN 2K 101—MATHEMATICS—I

(Common to all Branches)

[New Scheme]

Time : Three Hours | Maximum : 100 Marks

Answer **all** *questions.*

1. (a) Expand $e^{\cos x}$ upto the term containing x^4 using Maclaurin's series.

(b) If $x = r\cos\theta$ $y = r\sin\theta$, verify that $\dfrac{\partial(x, y)}{\partial(r, \theta)} \times \dfrac{\partial(r, \theta)}{\partial(x, y)} = 1$.

(c) Find $\dfrac{dy}{dx}$, when (i) $x^3 + y^3 = 3ax^2$ and (ii) $x^y + y^x = C$.

(d) Discuss the convergence or divegence of the series $\sum\limits_{n=1}^{\infty} \sin\left(\dfrac{1}{n}\right)$.

(e) Find the sum of the eigenvalues of the inverse of $A = \begin{bmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{bmatrix}$.

(f) Find the rank of matrix $\begin{bmatrix} 1 & 2 & -1 & -5 & 4 \\ 1 & 3 & -2 & -7 & 5 \\ 2 & -1 & 3 & 0 & 3 \end{bmatrix}$.

(g) Find the Fourier series of :

$$f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi, \text{ if } f(x) \end{cases}$$

is periodic with period 2π.

(h) Find half range sine series for $f(x) = x$ of periodicity $2l$ in the range $0 < x < l$.

(8 × 5 = 40 marks)

Turn over

2. (a) (i) Find the envelope of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, treating it as the envelope of its normals.

(8 marks)

(ii) If $u = f(x, y)$, where $x = r\cos\theta$ $y = r\sin\theta$, prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$.

(7 marks)

Or

(b) (i) Examine the function $f(x, y) = x^3 + y^3 - 3xy + 1$ for extreme value. (8 marks)

(ii) Show that the evolute of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is another cycloid.

(7 marks)

3. (a) (i) Test the convergence of $\sum_{n=1}^{\infty} \frac{n^3 + a}{2n + a}$. (8 marks)

(ii) If $y = (\sin^{-1} x)^2$, prove that $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - n^2 y_n = 0$. (7 marks)

Or

(b) (i) Test the convergence of the series :

$$\frac{1}{3}x + \frac{1.2}{3.5}x^2 + \frac{1.2.3}{3.5.7}x^3 + \ldots$$

(8 marks)

(ii) Test for convergence the following series :

$$\frac{2}{3.4} + \frac{2.4}{3.5.6} + \frac{2.4.6}{3.5.7.8} + \ldots \infty.$$

(7 marks)

4. (a) (i) Define :

1 Hermitian matrix.

2 Skew-Hermitian matrix.

3 Unitary Matrix.

4 Rank of a matrix with examples for each.

(8 marks)

(ii) Diagonalize :

$$\begin{bmatrix} -19 & 7 \\ -45 & 16 \end{bmatrix}.$$

(7 marks)

Or

(b) (i) Solve the following system of equations using Gauss elimination method :—

$$3x - y + z = 2$$
$$x + 5y + 2z = 6$$
$$2x + 3y + z = 0$$

(8 marks)

(ii) Find the eigenvalues of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. (7 marks)

5. (a) (i) Determine the constant term and the co-efficients of $\cos x$ and $\sin x$ in the Fourier series of the function given below :

x ...	30°	60°	90°	120°	150°	180°	240°	270°	300°	330°	360°
y ...	2.34	3.01	3.68	4.15	3.69	2.20	0.51	0.88	1.09	1.19	1.64

(8 marks)

(ii) Find half-range Fourier cosine series for $f(x) = x$ in $0 < x < \pi$.

(7 marks)

Or

(b) (i) Expand $f(x) = x^2, -\pi < x < \pi$ in that $f(x)$ is periodic with period 2π and hence deduce that :

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots = \frac{\pi^2}{12}.$$

(8 marks)

(ii) Find the half-range cosine series for the $f = f(x) = (x-1)^2$ in the interval $0 < x < 1$.

(7 marks)

[4 × 15 = 60 marks]