C 47642 **(Pages 3)** Name..................................

Reg. No..................................

SIXTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2008

CS 04 604—GRAPH THEORY AND COMBINATORICS

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

I. (a) Define planar graphs with suitable examples.

(b) Explain Chinese postman problem.

(c) Define eccentricity, radius and centre of a tree.

(d) Find the articulation points and biconnected components of the graph given below :

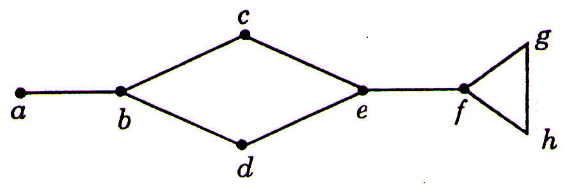


Fig. 1.

(e) For each integer $n > 0$.

Prove that :

(i) $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} = 2^n$

(ii) $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} + \dots + (-1)^n \binom{n}{n} = 0.$

(f) In how many ways can the letters in the word MISSISSIPPI be arranged ?

(g) Write the numeric function corresponding to the generating function $\dfrac{2 + 3z - 6z^2}{1 - 2z}$.

(h) Obtain the homogeneous solution of the recurrence relation $a_r - a_{r-1} - a_{r-2} = 0$, given $a_0 = 1, a_1 = 1$.

(8 × 5 = 40 marks)

Turn over

II. (a) (i) Show that a graph possess an Eulerian path if and only if it is connected and has either zero or two vertices of odd degree.

(8 marks)

(ii) Explain colouring of a graph with examples. (7 marks)

Or

(b) (i) Prove that a connected graph G remains connected after removing an edge e_i from G, if and and if e_i is in some circuit in G.

(8 marks)

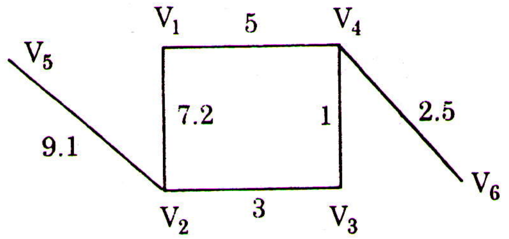
(ii) A graph of n vertices is a complete graph if and only if its chromatic polynomial is :

$$p_n(\lambda) = \lambda(\lambda - 1)(\lambda - 2) \dots (\lambda - n + 1).$$

(7 marks)

III. (a) (i) Show that a connected graph with n vertices and $(n - 1)$ edges is a tree. (8 marks)

(ii) Find the minimum spanning tree of the weighted graph given below :



(7 marks)

Or

(b) (i) Prove that a binary tree having 'n' vertices has $(n+1)/2$ pendant vertices. (8 marks)

(ii) Find the radius and diameter of the given tree :

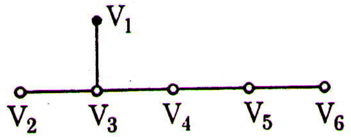


Fig. 2.

(7 marks)

IV. (a) (i) Determine the number of integers between 1 and 250 that are divisible by any of the integers 2, 3 an 5.

(8 marks)

JYOTHI ENGG. COLLEGE LIBRARY
THRISSUR EDUCATIONAL TRUST
C 47642
CHERUTHURUTHY

(ii) In how many ways can 6 men and 6 women be seated in a row :

1 If any person may sit next to any other.

2 If men and women must occupy alternate seat.

(7 marks)

Or

(b) (i) Among 100 students, 32 study mathematics, 20 study physics, 45 study biology, 15 study mathematics and biology, 7 study mathematics and physics, 10 study physics and biology, 30 do not study any of the three subjects.

1 Find the number of students studying all three subjects.

2 Find the number of students studying exactly one of the three subjects.

(8 marks)

(ii) Find the number of arrangements of the letters of the word "TALLAHASSEE". How many arrangements have no adjacent A's.

(7 marks)

V. (a) (i) Solve the recurrence relation

$$ar + 5ar - 1 + 6ar - 2 = 3r^2 - 2r + 1.$$

(8 marks)

(ii) Solve $ar = ar - 1 + ar - 2$ by the method of generating function. (7 marks)

Or

(b) (i) Solve the recurrence relation $ar + 6ar - 1 + 9ar - 2 = 3$ given that $a_0 = 0$ and $a_1 = 1$.

(8 marks)

(ii) Find the numeric function corresponding to the generating function $A(z) = \dfrac{(1+z)^2}{(1-z)^4}$.

(7 marks)

[4 × 15 = 60 marks]