**C 58398** (Pages : 3) Name.........................

Reg. No.........................



# SIXTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2009

## CS 04 604 GRAPH THEORY AND COMBINATORICS

(2004 Admissions)

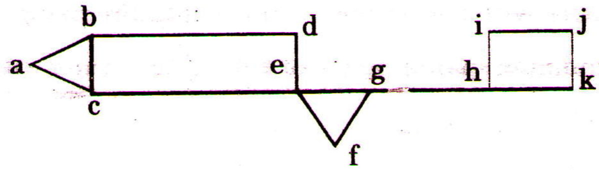
Time : Three Hours Maximum : 100 Marks

*Answer* **all** *questions.*

I. (a) Define Euler graph and Euler line with examples.

(b) Show that "If a graph has 2 vertices of odd degree, there must be a path joining these vertices.

(c) Find the articulation points and biconnected components for the following graph.




(d) Define the following :

(i) Minimum spanning tree.

(ii) Fundamental cut set.

(e) Three are three married couples and that A, B and C are Females and D, E, F are males. Arrange the six people around the table so that the sexes allernate.

(f) If '$x$' and '$y$' are variables and '$n$' is a positive integer then show that

$$(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} .$$

(g) Obtain the particular solution of the recurrence relation $ar + 5\ ar + 6\ ar - 2 = 3r^2$.

(h) Obtain the homogeneous solution of the recurrence relation

$4\ ar - 20\ ar - 1 + 17\ ar - 2 - 4\ ar - 3 = 0.$

(8 × 5 = 40 marks)

Turn over

II. (a) (i) Prove that if a connected graph G is decomposed into two subgraphs $g_1$ and $g_2$ there must be at least on vertex common between $g_1$ and $g_2$.

(8 marks)

(ii) Generate 2–isomorphic graphs for the following graph. Find out whether the two graphs have circuit correspondences or not. Justify your answer.

(7 marks)

*Or*

(b) (i) A graph has a dual if and only if it is planar. (8 marks)

(ii) If G = (V, E) be defined by V = ($V_1$, $V_2$, $V_3$, $V_4$) and E = { ($V_1$, $V_2$), ($V_1$, $V_3$), ($V_2$, $V_4$), ($V_3$, $V_4$)}. Use adjacency matrix to determine the number of paths of length 2 from $V_2$ to $V_3$.

(7 marks)

III. (a) (i) Prove that every tree with two or more vertices is 2 chromatic. (8 marks)

(ii) Explain how a minimum spanning tree of a weighted graph with real weights to edges is determined.

(7 marks)

*Or*

(b) (i) prove that "Every tree has either one or two centres". (8 marks)

(ii) Define rooled and binary trees with examples. (7 marks)

IV. (a) (i) Show that for all integers $n$, $r \geq 0$ and if $n + 1 > r$. $p(n+1, r) = \binom{n+1}{n+1-r} p(n, r)$.

(8 marks)

(ii) Determine the number of positive integers $n$, $1 \leq n \leq 200$ that are not divisible by 2, 3 or 5.

(7 marks)

*Or*

(b) (i) Find the total number of positive integers that can be formed From the digits 1, 2, 3, 4 if no digit is repeated in any one integer.

(ii) Thirty cars were assembled in a factory. The Options available were a radio, an air conditioner and white-wall tires. It is known that 15 of the cars have radios, 8 of them have air conditioners and 6 of them have white-wall tires Moreover, 3 of them have all three options. Determine.

(1) At most how many cars have one or more options ?

(2) At least how many cars do not have any options ?

V. (a) (i) Solve the recurrence relation $ar - 5\ ar - 1 + 6\ ar - 2 = 3^r + r$. (8 marks)

(ii) Obtain the numeric Function for the generating Function $A(z) = \frac{1}{1 - z^3}$ (7 marks)

*Or*

(b) (i) Solve the recurrence relation $ar - 2r - 1 = 7r^2$. (8 marks)

(ii) Solve $ar = 3ar - 1 + 2$ by the method of generating function. (7 marks)