C 58316 **(Pages : 3)** Name.........................

Reg. No........................

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SIXTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2009

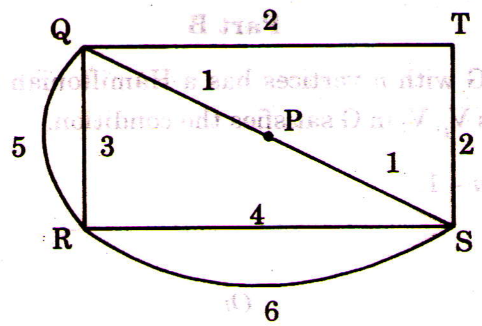
CS 2K 603—GRAPH THEORY AND COMBINATORICS

Time : Three Hours Maximum : 100 Marks

Part A

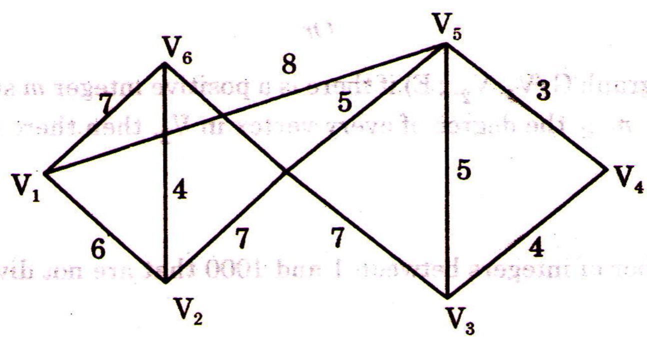
Answer **all** *questions.*

I. (a) Solve the Chinese postman problem for the weighted graph shown below.



(b) Explain how to compute chromatic number of a graph G, if chromatic Polynomial is given.

(c) Using Prim's algorithm, find a minimal spanning tree for the weighted graph shown below.



Turn over

(d) State the max-flow min-cut theorem.

(e) State fundamental Principles of counting. Using that, Prove

$nc_0 + nc_1 + nc_2 + ... nc_n = 2^n$.

(f) State the rules of Sum and Product.

(g) Find the recurrence relation and the initial condition for the sequence

0, 2, 6, 12, 20, 30, 42 ....

(h) Find the generating function for the sequences.

{1, –1, 1, –1, 1, –1, ... }

(8 × 5 = 40 marks)

Part B

II. (a) Prove that a graph G with n vertices has a Hamiltonian Path if the sum of the degrees of every pair of vertices V_i, V_j in G satisfies the condition.

$d(V_i) + d(V_j) \geq u - 1$.

(15 marks)

Or

(b) (i) Explain about platonic bodies. (7 marks)

(ii) Show that the complete graph K_n contains $\frac{1}{2}(n-1)!$ different Hamiltonian cycles.

(8 marks)

III. (a) Explain All pair shortest Path algorithm. (15 marks)

Or

(b) In a bipartite graph G (V_1, V_2, ; E) if there is a positive integer m such that the degree of every vector in $V_1 \geq m \geq$ the degree of every vertex in V_2, then there exists a complete matching from V_1 to V_2.

(15 marks)

IV. (a) Find the number of integers between 1 and 1000 that are not divisible by 2, 3, 5 or 7.

(15 marks)

Or

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(b) Determine the number of ways of placing $2t + 1$ in distinguishable balls in three distinct boxes so that two boxes together will contain more balls than the other one.

V. (a) (i) Solve the linear recurrence relation $c_n = 3c_{n-1} - 2c_{n-2}$ with $c_1 = 5, c_2 = 3$.

(7 marks)

(ii) Show that for any positive integer n, $(1 + n)^n$ is the exponential generating function for the sequence $p(n-0), p(n, 1), p(n, 2) \ldots p(n, n)\, 0, 0, 0 \ldots$

(8 marks)

Or

(b) In how many ways can 5 identical apples and 5 identical oranges be distributed among 5 people such that each person receives exactly 2 fruits ?

(15 marks)

[4 × 15 = 60 marks]