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Name..............................

Reg. No...........................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2009

EN 2K 102—MATHEMATICS—II

(Common to all Branches)

Time : Three Hours

Maximum : 100 Marks

Answer **all** *questions.*

I. (a) Solve $\left(x^2-4xy-2y^2\right)dx+\left(y^2-4xy-2x^2\right)dy=0$.

(b) Solve $\left(D^2-4D+13\right)y=e^{2x}$.

(c) Find $L^{-1}\left\{\log\frac{(s+1)}{(s-1)}\right\}$.

(d) Find $L\left[\int_0^t \frac{\sin u}{u}\,du\right]$.

(e) Prove that $\nabla\times\left(\phi\overline{A}\right)=\nabla\phi\times\overline{A}+\phi\left(\nabla\times\overline{A}\right)$.

(f) Find the directional derivative of $\phi = xy + yz + zx$ in the direction of the vector $\vec{i}+2\vec{j}+2\vec{k}$ at (1, 2, 0).

(g) Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F}=x^2\vec{i}+y^3\vec{j}$ and C is a portion of the parabola $y = x^2$ in the XY plane from A (0, 0) to B (1, 1).

(h) Use divergence theorem to evaluate $\iint_S \vec{F}\cdot\vec{n}\,dS$ where S is the surface of the rectangular parallelopiped $0\le x\le a, 0\le y\le b, 0\le z\le c$ and $\vec{F}=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$.

(8 × 5 = 40 marks)

II. (a) (i) Solve $\left(D^2+a\right)y=e^{3x}\left(x^2+1\right)$.

(7 marks)

(ii) Solve by method of variation of parameters, the differential equation $\frac{d^2y}{dx^2}+y=\sec x$.

(8 marks)

Or

Turn over

(b) (i) Find the orthogonal trajectories of a system of confocal and coaxial parabolas. (7 marks)

(ii) In a L-C-R circuit, the charge "q" on a plate of condenser is given by

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E \sin pt$$

the circuit is tuned to resonance. So that $p^2 = \frac{1}{LC}$. If initially, the current "i" and the charge q be zero, show that, for small values of R/L, the current in the circuit at time "t" is given by $\frac{ET}{2L}\sin pt$.

(8 marks)

III. (a) (i) Find the Laplace transform of $\frac{(1-\cos t)}{t^2}$. (7 marks)

(ii) Find the Laplace transform of the function $f(t) = \begin{cases} t, & \text{for } 0 < t < a \\ 2a - t, & \text{for } a < t < 2a. \end{cases}$ (8 marks)

Or

(b) (i) Find L $\{f(t)\}$ where $F(t) = \begin{cases} \sin wt & 0 < t < \pi/w \\ 0 & \pi/w < t < 2\pi/w \end{cases}$ if $F(t + 2\pi/w) = F(t)$ for all t.

(7 marks)

(ii) Solve $y'' + 5y' + 4y = x^2$. If $y'(0) = y(0) = 1$, using Laplace transform method. (8 marks)

IV. (a) (i) Find the angle between the surfaces $z = x^2 + y^2 - 3$ and $x^2 + y^2 + z^2 = 9$ at the point $(2, -1, 2)$.

(7 marks)

(ii) Find a and b so that the surfaces $ax^2 - byz = (a + 2)x$ and $4x^2y + z^3 = 4$ cut orthogonally at $(1, -1, 2)$.

(8 marks)

Or

(b) (i) Find the constant a, b, c. So that the vector $\vec{F} = (axy + bz^3)\vec{i} + (3x^2 - cz)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational.

(7 marks)

(ii) Prove that $\nabla(u \cdot v) = v \times \text{curl } u + u \times \text{curl } v + (v \cdot \nabla)u + (u \cdot \nabla)v$. (8 marks)

V. (a) Verify Green's theorem in the xy-plane for $\oint_C (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$ where C is the boundary of the region given by $x = y = 0$ and $x + y = 1$.

Or

(b) Verify divergence theorem for $\vec{F} = 4x\vec{i} + 2y^2\vec{j} + z^2\vec{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. (15 marks)

[4 × 15 = 60 marks]