C 58169 (Pages : 2) Name..............................

Reg. No..............................

FOURTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION JUNE 2009

EN 04 401 (A)—ENGINEERING MATHEMATICS—IV

(Common for all except CS and IT)

[2004 Admissions]

Time : Three Hours Maximum : 100 Marks

Part A

Answer **all** *questions.*

I. (a) Show that $f(z) = |z|^2$ is differentiable at $z = 0$ but not analytic at $z = 0$.

(b) An analytic function with constant modulus is constant.

(c) Evaluate the integral $\int_C \frac{z^2\, d^2}{2\text{-}3}$ where C is the unit circle $|z| = 1$.

(d) Expand $f(z) = \cos z$ in a Taylor's series about $z = 0$.

(e) Show that $P_n(1) = 1$ and $P_n(-1) = (-1)^n$.

(f) Show that $\frac{2}{5} P_3(x) + \frac{3}{5} P_1(x) = x^3$.

(g) Solve using separation of variable method $y\, u_x + x\, u_y = 0$.

(h) Find the image of $x + y = 2$ under the transformation $w = z^2$.

($8 \times 5 = 40$ marks)

Part B

II. (a) (i) If $\sin(\theta + i\varphi) = \cos\alpha + i \sin\alpha$ prove that $\cos^2\theta = \pm \sin\alpha$. (8 marks)

(ii) Show that the transformation $w = \frac{1}{z}$ transforms circles and straight lines in the z-plane into circles or straight lines in the w-plane.

(7 marks)

Or

(b) (i) Find the bilinear transformation that map the points $0, 1, \infty$ of the z-plane into $i, 1, -i$ of the w-plane.

(8 marks)

Turn over

(ii) Determine the analytic function $u + iv$ whose real part is $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ and verify whether u satisfies Laplace equation.

(7 marks)

III. (a) (i) If $f(a) = \int_C \frac{3z^2 + 7z + 1}{z - a} dz$, where C is $|z| = 2$, find $f(4)$, $f'(1)$ and $f''(1)$. (8 marks)

(ii) Find the Laurent's series for $f(z) = \frac{z}{(z^2 - 1)(z^2 + 4)}$ if $1 < |2| < 2$. (7 marks)

Or

(b) (i) Evaluate $\int_0^\infty \frac{x \sin mx}{x^2 + a^2} dx$ $m > 0$, $a > 0$ using contour integration. (8 marks)

(ii) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z - 1)^2 (z - 2)} dz$ around $|z| = 3$. (7 marks)

IV. (a) Find the series solution of the equation $4x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + y = 0$. (15 marks)

Or

(b) Show that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$. (15 marks)

V. (a) If a string of length l is initially at rest in equilibrium position and each point of it is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = v_0 \sin^3 \frac{\pi x}{l}$, $0 < x < l$, determine the transverse displacement $y(x, t)$.

(15 marks)

Or

(b) A rectangular plate with insulated surfaces is 8 cm wide and so long compared to its width that it may be considered as an infinite plate. If the temperature along short edge $y = 0$ is $u(x, 0) = 100 \sin \frac{\pi x}{8}$, $0 < x < 8$ while two long edges $x = 0$ and $x = 8$ as well as the other short edge are kept at 0°C. Find the steady state temperature at any point of the plate.

(15 marks)

[4 × 15 = 60 marks]