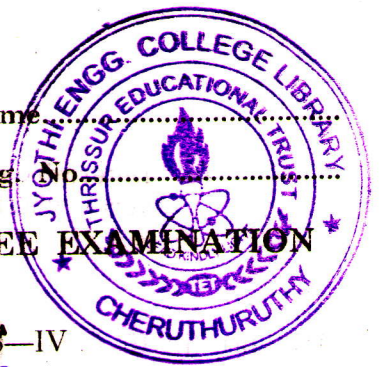


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(Pages : 2)

Name:

Reg. No:



FOURTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION
JUNE 2009

EN 04 401 (A)—ENGINEERING MATHEMATICS—IV

(Common for all except CS and IT) ER

[2004 Admissions]

Time : Three Hours

Maximum : 100 Marks

Part A

Answer all questions.

I. (a) Show that $f(z) = |z|^2$ is differentiable at $z = 0$ but not analytic at $z = 0$.

(b) An analytic function with constant modulus is constant.

(c) Evaluate the integral $\int_C \frac{z^2 dz}{2-z}$ where C is the unit circle $|z| = 1$.

(d) Expand $f(z) = \cos z$ in a Taylor's series about $z = 0$.

(e) Show that $P_n(1) = 1$ and $P_n(-1) = (-1)^n$.

(f) Show that $\frac{2}{5} P_3(x) + \frac{3}{5} P_1(x) = x^3$.

(g) Solve using separation of variable method $y u_x + x u_y = 0$.

(h) Find the image of $x + y = 2$ under the transformation $w = z^2$.

(8 × 5 = 40 marks)

Part B

II. (a) (i) If $\sin(\theta + i\phi) = \cos \alpha + i \sin \alpha$ prove that $\cos^2 \theta = \pm \sin \alpha$.

(8 marks)

(ii) Show that the transformation $w = \frac{1}{z}$ transforms circles and straight lines in the z -plane into circles or straight lines in the w -plane.

(7 marks)

Or

(b) (i) Find the bilinear transformation that map the points $0, 1, \infty$ of the z -plane into $i, 1, -i$ of the w -plane.

(8 marks)

Turn over

- (ii) Determine the analytic function $u + iv$ whose real part is $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ and verify whether u satisfies Laplace equation.

(7 marks)

- III. (a) (i) If $f(z) = \int_C \frac{3z^2 + 7z + 1}{z - a} dz$, where C is $|z| = 2$, find $f(4)$, $f'(1)$ and $f''(1)$.

(8 marks)

- (ii) Find the Laurent's series for $f(z) = \frac{z}{(z^2 - 1)(z^2 + 4)}$ if $1 < |z| < 2$.

(7 marks)

Or

- (b) (i) Evaluate $\int_0^\infty \frac{x \sin mx}{x^2 + a^2} dx$ $m > 0, a > 0$ using contour integration.

(8 marks)

- (ii) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z - 1)^2 (z - 2)} dz$ around $|z| = 3$.

(7 marks)

- IV. (a) Find the series solution of the equation $4x \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = 0$.

(15 marks)

Or

- (b) Show that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$.

(15 marks)

- V. (a) If a string of length l is initially at rest in equilibrium position and each point of it is given the

velocity $\left(\frac{\partial y}{\partial t} \right)_{t=0} = v_0 \sin^3 \frac{\pi x}{l}$, $0 < x < l$, determine the transverse displacement $y(x, t)$.

(15 marks)

Or

- (b) A rectangular plate with insulated surfaces is 8 cm wide and so long compared to its width that it may be considered as an infinite plate. If the temperature along short edge $y = 0$ is

$u(x, 0) = 100 \sin \frac{\pi x}{8}$, $0 < x < 8$ while two long edges $x = 0$ and $x = 8$ as well as the other short edge are kept at 0°C . Find the steady state temperature at any point of the plate.

(15 marks)

[4 × 15 = 60 marks]