

C 57541

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Name.....

Reg. No.....

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING)  
DEGREE EXAMINATION, JUNE 2009

EN 2K 101—MATHEMATICS—I

(Common to all Branches)

Time : Three Hours

Maximum : 100 Marks

Answer all questions.

- I. (a) Evaluate  $\lim_{x \rightarrow 0} \frac{\tan x}{x}$ .
- (b) Expand  $e^x$  using Taylor's series around  $x = 1$ .
- (c) Find the error in the calculation of the volume of a right-circular cone if there is a shortage of 0.01 cm. per cm. in the measure used. The radius and the height of the cone are 4 cm. and 6 cm. respectively.

(d) Find the  $n$ th derivative of  $\frac{1}{x^2 + 3x + 2}$ .

(e) Find the rank of the matrix  $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ .

(f) Find the eigenvalues and eigenvectors of  $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ .

(g) Find the Fourier series of  $f(x) = x + x^2, -\pi < x < \pi$ .

(h) Find the half range cosine series for  $f(x) = \begin{cases} 1, 0 \leq x \leq \frac{a}{2} \\ -1, \frac{a}{2} < x < a. \end{cases}$

(8 × 5 = 40 marks)

II. (a) (i) Find the radius of curvature of  $4ay^2 = (2a - x)^3$  at  $(a, a/2)$ .

(8 marks)

(ii) If  $u = \sin^{-1} \left( \frac{x^2 + y^2}{x + y} \right)$ , prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$ .

(7 marks)

Or

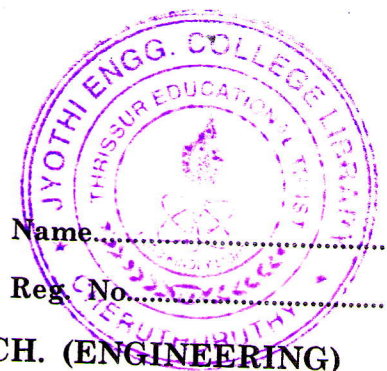
(b) (i) Find the maximum and minimum values of  $f(x, y) = y^2 + 4xy + 3x^2 + x^3$ .

(8 marks)

(ii) Find the evaluate of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .

(7 marks)

Turn over



EE

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III. (a) (i) Test the convergence  $\frac{1}{\sqrt{1+\sqrt{2}}} + \frac{1}{\sqrt{2+\sqrt{3}}} + \frac{1}{\sqrt{3+\sqrt{4}}} + \dots$ . (8 marks)

(ii) If  $y = e^{a \sin^{-1} x}$ , prove that  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$ . (7 marks)

Or

(b) (i) Test for convergence  $2 - \frac{3}{2} + \frac{4}{3} - \frac{5}{4} + \dots$ . (8 marks)

(ii) Discuss the convergence of the series  $\frac{1}{3} + \frac{1.2}{3.5} + \frac{1.2.3}{3.5.7} + \dots$ . (7 marks)

IV. (a) (i) Show that the equations  $2x + 6y + 11 = 0$ ;  $6x + 20y - 6z + 3 = 0$  and  $6y - 18z + 1 = 0$  are not consistent. (8 marks)

(ii) Find the eigen values of  $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ . (7 marks)

Or

(b) (i) Solve the following equations by matrix inversion method :

$$x + 2y - z = 2 ; 3x + 8y + 2z = 10 ; 4x + 9y - z = 12.$$

(8 marks)

(ii) Verify Cayley-Hamilton theorem for  $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ . (7 marks)

V. (a) (i) Expand  $f(x) = x \cos x$  in  $-\pi < x < \pi$  in Fourier series. (8 marks)

(ii) Expand  $f(x) = \begin{cases} x^2; & 0 < x < 1 \\ 2-x; & 1 < x < 2 \end{cases}$  as a cosine series. (7 marks)

Or

(b) (i) Obtain the Fourier series for the function  $f(x) = x^2 - 2$  in  $-2 < x < 2$ . (8 marks)

(ii) Find a half range sine series for  $f(x) = \cos x$  in  $0 < x < \pi$ . (7 marks)

[4 × 15 = 60 marks]