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Name..............................

Reg. No..............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2009

EN 2K 101—MATHEMATICS—I

(Common to all Branches)

Time : Three Hours

Maximum : 100 Marks

Answer **all** *questions.*

I. (a) Evaluate $\lim_{x \to 0} \frac{\tan x}{x}$.

(b) Expand e^x using Taylors series around $x = 1$.

(c) Find the error in the calculation of the volume of a right-circular cone if there is a shortage of 0.01 cm. per cm. in the measure used. The radius and the height of the cone are 4 cm. and 6 cm. respectively.

(d) Find the nth derivative of $\frac{1}{x^2 + 3x + 2}$.

(e) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$.

(f) Find the eigenvalues and eigenvectors of $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

(g) Find the Fourier series of $f(x) = x + x^2, -\pi < x < \pi$.

(h) Find the half range cosine series for $f(x) = \begin{cases} 1, 0 \le x \le \frac{a}{2} \\ -1, \frac{a}{2} < x < a. \end{cases}$

(8 × 5 = 40 marks)

II. (a) (i) Find the radius of curvature of $4ay^2 = (2a - x)^3$ at $(a, a/2)$. (8 marks)

(ii) If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. (7 marks)

Or

(b) (i) Find the maximum and minimum values of $f(x, y) = y^2 + 4xy + 3x^2 + x^3$. (8 marks)

(ii) Find the evaluate of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (7 marks)

Turn over

III. (a) (i) Test the convergence $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots$. (8 marks)

(ii) If $y = e^{a\sin^{-1}x}$, prove that $\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}-\left(n^2+a^2\right)y_n = 0$. (7 marks)

Or

(b) (i) Test for convergence $2-\frac{3}{2}+\frac{4}{3}-\frac{5}{4}+\cdots$. (8 marks)

(ii) Discuss the convergence of the series $\frac{1}{3}+\frac{1.2}{3.5}+\frac{1.2.3}{3.5.7}+\cdots$. (7 marks)

IV. (a) (i) Show that the equations $2x + 6y + 11 = 0$; $6x + 20y - 6z + 3 = 0$ and $6y - 18z + 1 = 0$ are not consistent.

(8 marks)

(ii) Find the eigen values of $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. (7 marks)

Or

(b) (i) Solve the following equations by matrix inversion method :

$x + 2y - z = 2$; $3x + 8y + 2z = 10$; $4x + 9y - z = 12$.

(8 marks)

(ii) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. (7 marks)

V. (a) (i) Expand $f(x) = x\cos x$ in $-\pi < x < \pi$ in Fourier series. (8 marks)

(ii) Expand $f(x) = \begin{cases} x^2; & 0 < x < 1 \\ 2-x; & 1 < x < 2 \end{cases}$ as a cosine series. (7 marks)

Or

(b) (i) Obtain the Fourier series for the function $f(x) = x^2 - 2$ in $-2 < x < 2$. (8 marks)

(ii) Find a half range sine series for $f(x) = \cos x$ in $0 < x < \pi$. (7 marks)

[4 × 15 = 60 marks]