C 58170 (Pages : 3) Name.........................

Reg. No.........................

FOURTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION JUNE 2009

EN 04 401(B)—ENGINEERING MATHEMATICS—IV

(Common for CS/IT)

Time : Three Hours Maximum : 100 Marks

Part A

Answer **all** *questions.*

1. Find the MGF of the binomial distribution and hence find its mean and variance.
2. A line of length 'a' units is divided into 2 parts. If the first part is of length X, find E(X), Var (X) and E{X(a – X)}.
3. Show that if a most efficient estimator A and a less efficient estimator B with efficiency e tend to joint normality for large samples, B – A tends to zero correlation with A.
4. State as precisely as possible the properties of the Maximal Likelihood Estimate.
5. Define Producer's risk and Consumer's risk.
6. State the properties of t-distribution.
7. A radioactive source emits particles at a rate of 5 per minute in accordance with Poisson Process. Each particle emitted has a probability 0.6 of being recorded. Find the probability that 10 particles are record in 4-min period.
8. State and prove Chapman-Kolmogorov theorem.

(8 × 5 = 40 marks)

Part B

9. (a) If m things are distributed among 'a' men and 'b' women, show that the probability that the number of things received by men is odd, is $\frac{1}{2}\left[\frac{(b+a)^m-(b-a)^m}{(b+a)^m}\right]$.

(8 marks)

(b) Show that in a Poisson distribution with unit mean, mean deviation about mean is ($2/e$) times the standard deviation.

(7 marks)

Or

(c) The mean yield for one-acre plot is 662 kilos with a s.d. 32 kilos. Assuming normal distribution, how many one-acre plots in a batch of 1,000 plots would you expect to have yield (i) over 700 kilos ; (ii) below 650 kilos ; (iii) what is the lowest yield of the best 100 plots.

(8 marks)

Turn over

(d) A car hire firm has two care which it hires out day by day. The number of demands for a car on each day is distributed as Poisson variate with mean 1.5. Calculate the proportion of days on which (i) neither car is used and (ii) some demand is refused.

(7 marks)

10. (a) A sample of n independent observations is drawn from the rectangular population

$$f(x,\beta) = \begin{cases} \frac{1}{\beta}, & 0 < x \leq \beta, 0 < \beta < \infty \\ 0, & \text{elsewhere} \end{cases}$$

Find Maximum Likelibood Estimate for β.

(8 marks)

(b) Let $x_1, x_2, \ldots x_n$ be a random sample from $N(\mu, \sigma^2)$ population. Find sufficient estimators for μ and σ^2.

(7 marks)

Or

(c) A random variable X has the probability density function :

$$f(x) = (\beta+1)x^{\beta} \text{ for } (0 < x < 1), (\beta \neq -1)$$
$$= 0, \quad \text{otherwise}$$

Based on n-independent observations on X, obtain the maximum likelihood estimator of β and an unbiased estimator of $(\beta+1)/(\beta+2)$ when $\beta \neq -2$.

(8 marks)

(d) Obtain the minimum variance unbiased estimates for μ in the normal population $N(\mu, \sigma^2)$ where σ^2 is known.

(7 marks)

11. (a) The table given below shows the results of a survey in which 250 respondents were classified according to levels of education and attitude towards student's agitation in a certain town. Test whether 2 criteria of classification are independent.

Attitude

Education		Against	Neutral	For
Middle school	...	40	25	5
High school	...	40	20	5
College	...	30	15	30
Post Graduate	...	15	15	10

(15 marks)

Or

(b) Two samples drawn from two different population gave the following results :

	Size	*Mean*	*S.D.*
Sample I ...	400	124	14
Sample II ...	250	120	12

Find the 95% confidence limits for the difference of the population means.

(8 marks)

(c) Theory predicts that the proportion of beans in four groups A, B, C, D should be 9 : 3 : 3 : 1. In an experiment among 1600 beans, the numbers in the four groups were 882, 313, 287 and 118. Does the experiment support the theory ?

(7 marks)

12. (a) A gambler has Rs. 2. He bets Re. 1 at a time and wins Re. 1 with probability 1/2. He stops playing if he loses Rs. 2 or wins Rs. 4 (i) What is the transition probability matrix of the related Markov chain ? (ii) What is the probability that he lost his money at the end of 5 plays ? (iii) What is the probability that the game lasts more than 7 plays ?

(15 marks)

Or

(b) The number of accidents in a city follows a Poisson process with a mean of 2 per day and the number X_i ; of people involved in the i^{th} accident has the distribution (independent)

$P\{X_i = K\}\frac{1}{2^k}(K \geq 1)$. Find the mean and variance of the number of people involved in accidents per week.

(15 marks)

[4 × 15 = 60 marks]