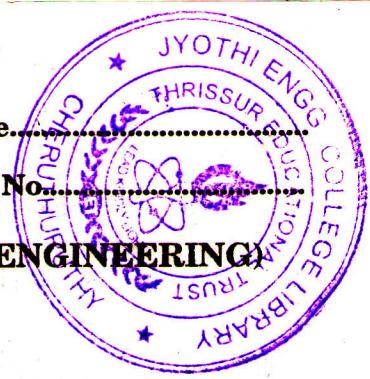


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Name.....

Reg. No.....



**COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING)  
DEGREE EXAMINATION, JUNE 2009**

**Mathematics**

**EN 04—102 MATHEMATICS-II**

(2004 admissions)

Time : Three Hours

Maximum : 100 Marks

**Part A**

- I. 1 Solve  $(x^2 - y^2) dx - xydy = 0$ .
- 2 Solve  $\frac{dy}{dx} = (4x + y + 1)^2$ , if  $y(0) = 1$ .
- 3 Find the Laplace transform of  $\sin^3 2t$ .
- 4 Apply convolution theorem to evaluate  $L^{-1} \left\{ \frac{s}{(s^2 + a^2)^2} \right\}$ .
- 5 Find  $\text{Curl} (e^{xyz} (i + j + k))$ .
- 6 If  $u = x^2 + y^2 + z^2$  and  $\mathbf{V} = xi + yj + zk$ , show that  $\text{div} (u \mathbf{V}) = 5u$ .
- 7 Evaluate  $\int_S (yzi + zxj + xyk) \cdot d\mathbf{s}$ , where S is the surface of the sphere  $x^2 + y^2 + z^2 = a^2$  in the first octant.
- 8 If  $\mathbf{F} = 3xyi - y^2j$ , evaluate  $\int_C \mathbf{F} \cdot d\mathbf{R}$ , where C is the curve in the xy-plane  $y = 2x^2$  from  $(0, 0)$  to  $(1, 2)$ .

(8 × 5 = 40 marks)

**Part B**

- II. (a) (i) Solve  $(D^2 - 1)y = x \sin 3x + \cos x$ . (8 marks)
- (ii) Solve  $\frac{dz}{dx} + \left( \frac{z}{x} \right) \log z = \frac{z}{x} (\log z)^2$ . (7 marks)

*Or*

**Turn over**

(b) (i) Solve  $xy(1+xy^2)\frac{dy}{dx} = 1$ . (8 marks)

(ii) Solve  $(D^4 + 2D^2 + 1)y = x^2 \cos x$ . (7 marks)

III. (a) (i) Use Laplace transform to solve  $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$  with  $x=2, \frac{dx}{dt}=-1$  at  $t=0$ . (8 marks)

(ii) Find the inverse Laplace transform of  $\frac{1}{s(s+a)^3}$ . (7 marks)

Or

(b) (i) Find the Laplace transform of  $\int_0^t \frac{e^t \sin t}{t} dt$ . (8 marks)

(ii) Find the inverse Laplace transform of  $\frac{s}{s^4 + 4a^4}$ . (7 marks)

IV. (a) (i) Show that  $\operatorname{div}(\operatorname{grad} r^n) = n(n+1)r^{n-2}$  where  $r^2 = x^2 + y^2 + z^2$ . (8 marks)

(ii) If  $u = x^2yz, V = xy - 3z^2$ , find  $\nabla(\nabla u \cdot \nabla V)$ . (7 marks)

Or

(b) (i) If  $u = x^2 + y^2 + z^2$  and  $V = xi + yj + zk$ , show that  $\operatorname{div}(uV) = 5u$ . (8 marks)

(ii) If  $A$  is a constant vector and  $R = xi + yj + zk$ , prove that  $\operatorname{curl}(A \times R) = 2A$ . (7 marks)

V. (a) Evaluate  $\int_S \vec{F} \cdot d\vec{s}$ , where  $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$  S is the surface bounding the region

$$x^2 + y^2 = 4, z = 0 \text{ and } z = 3.$$

(8 marks)

(b) If  $\vec{r} = xi + yj + zk$ , show that  $\nabla \cdot \vec{r} = 3$  and  $\nabla \times \vec{r} = 0$ . (7 marks)

Or

(ii) (a) If  $\vec{V} = \frac{xi + yj + z^k}{\sqrt{x^2 + y^2 + z^2}}$ , show that

$$\nabla \cdot \vec{V} = \frac{2}{\sqrt{x^2 + y^2 + z^2}} \text{ and } \nabla \times \vec{V} = 0.$$

(8 marks)

(b) Evaluate the line integral  $\int_C [(x^2 + xy)dx + (x^2 + y^2)dy]$ , where  $C$  is the square formed by the lines  $y = \pm 1, x = \pm 1$ .

(7 marks)

[4 × 15 = 60 marks]