C 57512 (Pages : 2) Name.............................

Reg. No.............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2009

Mathematics

EN 04—102 MATHEMATICS—II

(2004 admissions)

Time : Three Hours Maximum : 100 Marks

Part A

I. 1 Solve $\left(x^2 - y^2\right)dx - xydy = 0$.

2 Solve $\frac{dy}{dx} = (4x + y + 1)^2$, if $y(0) = 1$.

3 Find the Laplace transform of $\sin^3 2t$.

4 Apply convolution theorem to evaluate $L^{-1} + \left(\frac{s}{\left(s^2 + a^2\right)^2}\right)$.

5 Find Curl $\left(e^{xyz}\left(i + j + k\right)\right)$.

6 If $u = x^2 + y^2 + z^2$ and $V = xi + yj + z^k$, show that div $(u\text{ V}) = 5u$.

7 Evaluate $\int_S (yzi + zxj + xyk).ds$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant.

8 If $F = 3xyi - y^2 j$, evaluate $\int F.dR$, where C is the curve in the xy-plane $y = 2x^2$ from $(0, 0)$ to $(1, 2)$.

(8 × 5 = 40 marks)

Part B

II. (a) (i) Solve $\left(D^2 - 1\right)y = x \sin 3x + \cos x$. (8 marks)

(ii) Solve $\frac{dz}{dx} + \left(\frac{z}{x}\right)\log z = \frac{z}{x}(\log z)^2$. (7 marks)

Or

Turn over

(b) (i) Solve $xy\left(1+xy^2\right)\frac{dy}{dx}=1.$ (8 marks)

(ii) Solve $\left(D^4+2D^2+1\right)y=x^2\cos x.$ (7 marks)

III. (a) (i) Use Laplace transform to solve $\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^t$ with $x=2, \frac{dx}{dt}=-1$ at $t=0$. (8 marks)

(ii) Find the inverse Laplace transform of $\frac{1}{s(s+a)^3}$. (7 marks)

Or

(b) (i) Find the Laplace transform of $\int_0^t \frac{e^t \sin t}{t}\,dt.$ (8 marks)

(ii) Find the inverse Laplace transform of $\frac{s}{s^4+4a^4}$. (7 marks)

IV. (a) (i) Show that div (grad r^n) = $n(n+1)r^{n-2}$ where $r^2=x^2+y^2+z^2$. (8 marks)

(ii) If $u=x^2yz, V=xy-3z^2$, find $\nabla(\nabla u.\nabla V)$. (7 marks)

Or

(b) (i) If $u=x^2+y^2+z^2$ and $V=xi+yj+z^k$, show that div $(uV)=5u$. (8 marks)

(ii) If A is a constant vector and $R=xi+yj+z^k$, prove that curl $(A\times R)=2A$. (7 marks)

V. (i) (a) Evaluate $\int_S \vec{F}.d\vec{s}$, where $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ S is the surface bounding the region $x^2+y^2=4, z=0$ and $z=3$.

(8 marks)

(b) If $\vec{r}=xi+yj+zk$, show that $\nabla\cdot r=3$ *and* $\nabla\times\vec{r}=0$. (7 marks)

Or

(ii) (a) If $\vec{V}=\frac{xi+yj+z^K}{\sqrt{x^2+y^2+z^2}}$, show that

$$\nabla.\vec{V}=\frac{2}{\sqrt{x^2+y^2+z^2}} \text{ and } \nabla\times\vec{V}=0.$$

(8 marks)

(b) Evaluate the line integral $\int_C \left[\left(x^2+xy\right)dx+\left(x^2+y^2\right)dy\right]$, where C is the square formed by the lines $y=\pm1, x=\pm1$.

(7 marks)

[4 × 15 = 60 marks]