C 57511 (Pages : 3) Name..............................

Reg. No..............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2009

Mathematics

EN 04—101 MATHEMATICS—I

(2004 Admissions)

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions in Part A and* **one** *full question from each Unit in Part B.*

Part A

I. (a) Evaluate

$$\operatorname*{Lt}_{x\to 0} \frac{\log \sec x - \frac{1}{2}x^2}{x^4}.$$

(b) If $u = \tan^{-1}\left(\frac{y}{x}\right)$ where $x = e^t - e^{-t}$, and $y = e^t + e^{-t}$, find $\frac{du}{dt}$.

(c) A ball is dropped from a height h metres. Each time the ball hits the ground, it rebounds a distance r times the distance fallen where $0 < r < 1$. If $h = 3$ metres and $r = \frac{2}{3}$, find the total distance travelled by the ball.

(d) Find the n^{th} derivative of $x^2 \log 3x$.

(e) If the augmented matrix of a system of equations is equivalent to $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & -3 & -1 & -2 \\ 0 & 0 & \lambda-8 & \mu-11 \end{bmatrix}$, find the values of λ and μ for which the system has no solution.

(f) If 2 is an eigenvalue of $\begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$ find the other two.

(g) If $f(x) = x^2$ in $-2 \le x \le 2$, find the values of a_0 and a_n.

Turn over

(h) If $\frac{a_0}{2} + \Sigma a_n \cos \frac{n\pi x}{l}$ is the half-range cosine series of $f(x)$ of period $2l$ in $(0,l)$, then show that the mean square value of $f(x)$ in $(0,l)$, is $\frac{l}{2}\left(\frac{a_0^2}{2} + \Sigma a_n^2\right)$.

(8 × 5 = 40 marks)

Part B

II. (a) (i) If the centre of curvature of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at one end of the minor axis lies at the other end, then show that the eccentricity of the ellipse is $\frac{1}{\sqrt{2}}$.

(7 marks)

(ii) Devide 24 into three parts such that the continued product of the first square of the second and the cube of the third may be maximum.

(8 marks)

Or

(b) (i) If $x = u + v + w$, $y = vw + wu + uv$, $z = uvw$ and F is a function of x, y, z, show that

$$u\frac{\partial F}{\partial u} + v\frac{\partial F}{\partial v} + w\frac{\partial F}{\partial w} = x\frac{\partial F}{\partial x} + 2y\frac{\partial F}{\partial y} + 3z\frac{\partial F}{\partial t}.$$

(7 marks)

(ii) Find the maximum and minimum values of

$$x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$$

(8 marks)

III. (a) (i) Test the convergence of the series

$$\Sigma \frac{1}{\sqrt{n} + \sqrt{n+1}}.$$

(7 marks)

(ii) Using Taylor's Theorem, express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x - 1)$.

(8 marks)

Or

(b) (i) State the values of x for which the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$ converge. (7 marks)

(ii) Using Maclaurin's series expand the function $\log(1 + x)$. Hence deduce that

$$\log\sqrt{\frac{1+x}{1-x}} = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots$$

(8 marks)

IV. (a) (i) By finding A^{-1}, solve the linear equation AX = B, where

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 1 & 2 & 0 \\ 5 & 1 & 1 \end{bmatrix}, \; X = \begin{bmatrix} x \\ y \\ Z \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix}.$$

(7 marks)

(ii) Using Cayley-Hamilton theorem, find A^{-1} when $A = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}$. (8 marks)

Or

(b) Find the eigenvalues, eigenvectors and the modal matrix of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ and hence reduce the quadratic form $x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2\,x_3$ to a canonical form. Also state its nature.

(15 marks)

V. (a) (i) Develop $f(x)$ in Fourier series in the interval (–2, 2), if

$$f(x) = 0, \quad -2 < x < 0$$

$$= 1, \quad 0 < x < 2.$$

(7 marks)

(ii) Obtain cosine and sine series for $f(x) = x$, in the interval $0 \le x \le \pi$. Hence show that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}.$$

(8 marks)

Or

(b) Determine the first two harmonics of the Fourier series for the following values :—

x° :	30	60	90	120	150	180	210	240	270	300	330	360
y :	2.34	3.01	3.68	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64

(15 marks)