Reg No.:_______________ 00000MA101121804 Name:_______________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech examinations (S) September 2020 S1/S2 (2015 Scheme)

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks. Marks

1	a)	Determine whether the series $\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^{k+2}$ converges. If so, find the sum	(2)
	b)	Find the Maclaurin series expansion of $f(x) = \boldsymbol{ln(1-x)}$ up to 3 terms	(3)
2	a)	Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $ye^x - 5cos2z = 3z$	(2)
	b)	Use chain rule to find $\frac{dw}{dx}$ at (0,1,2) for $w = xy + yz,\ \ y = \sin x, z = e^x$.	(3)
3	a)	Find the velocity of a particle moving along the curve $\bar{r}(t) == e^t \sin t\,\bar{i} + e^t \cos t\,\bar{j} + t\,\bar{k}$ at t = π	(2)
	b)	Find the unit normal to the surface $yz + zx + xy = c$ at (-1,2,3)	(3)
4	a)	Evaluate $\int_1^2 \int_y^{3-y} dx\, dy$	(2)
	b)	Evaluate $\int_1^2 \int_0^x \frac{dy\,dx}{x^2+y^2}$.	(3)
5	a)	Find the value of constant a so that if $\bar{F} = (3x - 2y + z)\,\boldsymbol{i} + (4x - ay + z)\boldsymbol{j} + (x - y + 2z)\boldsymbol{k}$ is solenoidal.	(2)
	b)	Find the work done by a force field $F(x,y) = -y\boldsymbol{i} + x\boldsymbol{j}$ acting on a particle moving along the circle $x^2 + y^2 = 3$ from $(\sqrt{3}, 0)$ to $(0, \sqrt{3})$	(3)
6	a)	Determine the source and sink of the vector field $F(x,y,z) = 2(x^3 - 2x)\boldsymbol{i} + 2(y^3 - 2y)\boldsymbol{j} + 2(z^3 - 2z)\boldsymbol{k}$	(2)
	b)	Using Stoke's theorem prove that $\int_C \bar{r}\,.\,d\bar{r} = 0$ where $\bar{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$ and C is any closed curve.	(3)

PART B

Module 1

Answer any two questions, each carries 5 marks.

7 Test the convergence of the infinite series $\sum_{k=1}^{\infty}\left(\frac{k}{k+1}\right)^{k^2}$. (5)

8 Examine the convergence of $\sum_{k=0}^{\infty}\frac{(k+4)!}{4!k!4^k}$ (5)

9 Find the radius of convergence of the power series $\sum_{k=1}^{\infty}\frac{(2x-3)^k}{4^{2k}}$ (5)

Module 1I

Answer any two questions, each carries 5 marks.

10 The height and radius of a circular cone is measured with errors of atmost 3% and 5% respectively. Use differentials to approximate the maximum percentage error in calculated volume. (5)

11 If $u = x^2tan^{-1}\left(\frac{y}{x}\right) - y^2tan^{-1}\left(\frac{x}{y}\right)$, find $\frac{\partial^2u}{\partial x\partial y}$ (5)

12 Find relative extrema and saddle points, if any, of the function $f(x,y) = x^3 + y^3 - 15xy$. (5)

Module 1II

Answer any two questions, each carries 5 marks.

13 Find where the tangent line to the curve $\boldsymbol{r}(t) = e^{-2t}i + \cos t\, j + 3\sin t\, k$ at the point (1,1,0) intersects the YZ plane. (5)

14 Find the position and velocity vectors of the particle given (5)

a(t)=$(t+1)^{-2}$**j** -$e^{-2t}k$, **v**(0) = 3**i**-**j** , r(0)= k

15 A particle moves along a curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$ where t is the time. Find the component of acceleration at time t = 1 in the direction of $\vec{i} - 3\vec{j} + 2\vec{k}$ (5)

Module 1V

Answer any two questions, each carries 5 marks.

16 Evaluate $\iiint_R xysin\, z\, dV$ where R is the rectangular box defined by (5)

$0 \le x \le 1, 0 \le y \le 1, \quad 0 \le z \le \frac{\pi}{6}$

17 Sketch the region of integration and evaluate $\int_1^2\int_y^{y^2} dx\, dy$ by changing the order of integration. (5)

18 Use double integral to find the area bounded by the x – axis (5)

$y = 2x \;\; and \;\; x + y = 1$

Module V

Answer any three questions, each carries 5 marks.

19 Prove that $\int_C (x^2 - yz)\bar{\imath} + (y^2 - zx)\bar{\jmath} + (z^2 - xy)\bar{k}\ .\ d\bar{r}$ is independent of the path and evaluate the integral along any curve from (0,0,0) to (1,2,3). (5)

20 Evaluate $\int_C xy^2\, dx + xy\, dy$ where C is a triangle with vertices at $(0,0), (0,1)$ and (2,1) (5)

21 Evaluate $\int_C 2xy\, dx + (x^2 + y^2)dy$ along the curve $C: x = \cos t, y = \sin t$, $0 \le t \le \frac{\pi}{2}$ (5)

22 Determine whether $F(x, y) = 6y^2\, i + 12xy\, j$ is a conservative vector field. If so find the potential function for it. (5)

23 If$\bar{F} = (sinz + y\cos x)\boldsymbol{i} + (\sin x + 2\cos y)\boldsymbol{j} + (\sin y + x\cos z)\boldsymbol{k}$,find $Div\, \bar{F}$ and $Curl\, \bar{F}$. (5)

Module VI

Answer any three questions, each carries 5 marks.

24 Using Stoke's theorem, evaluate $\int_C \bar{F}\ .\ d\bar{r}$ where C is the boundary of the projection of the sphere$x^2 + y^2 + z^2 = 1$ on the XY plane with (5)

$$\bar{F} = (2x - y)\bar{\imath} - yz^2\bar{\jmath} - y^2 z\,\bar{k}$$

25 Using Green's theorem evaluate $\int_C (y^2 - 7y)dx + (2xy + 2x)\, dy$ where C is the circle $x^2 + y^2 = 1$ (5)

26 Evaluate using divergence theorem for $\vec{F} = x^2 i + zj + yzk$ taken over the cube bounded by $x = 0,\ x = 1, y = 0,\ y = 1,\ \ z = 0\ and\ z = 1$ (5)

27 Evaluate the surface integral $\iint_\sigma z^2\, ds$, where σ is the portion of the curve $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$ (5)

28 Use Green' theorem to find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (5)

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