Reg No.:_____   02000MA204052001 Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
Fourth semester B.Tech examinations (S), September 2020

Course Code: MA204
Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS
(AE, EC)

Max. Marks: 100   Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A
Answer any two questions

1 a) Let X be a random variable taking values $-1, 0$ and 1 such that $P(X = -1) = 2P(X = 0) = P(X = 1)$. Find (i) the pdf of X (ii) the mean of $(2X - 5)$.   7

   b) A pair of dice is thrown 5 times. If getting a doublet is considered to be a success, use Binomial distribution to find the probability of getting (i) atleast 2 successes (ii) at most 2 successes (iii) exactly 2 failures.   8

2 a) If $f(x) = kx^2 e^{-x}, x > 0$, find k, mean and variance of the random variable.   7

   b) The length of time (in minutes) a person speaks over the phone follows an exponential distribution with mean 4. Find the probability that the person will talk for (i) more than 8 minutes (ii) between 3 and 6 minutes (iii) less than 2 minutes.   8

3 a) Fit a Poisson distribution to the following data: 7

x	0	1	2	3	4	5	6
f	167	70	35	17	7	3	1

   b) The marks obtained by students in an intelligence test follow normal distribution with mean 45 and standard deviation 25. Find the percentage of students who scored marks (i) more than 80 (ii) between 30 and 70 (iii) below 35   8

PART B
Answer any two questions

4 a) A random sample of size 200 is taken from a population whose mean is 50 and variance is 600. Using Central Limit theorem, find the probability that the mean of the sample $\overline{X}$ will not differ from $\mu = 50$ by more than 5.   7

   b) The joint pdf of X, Y is given by $f(x, y) = k(x + 2y), x = 1,2,3; y = 1,2,3$. Find (i) k (ii) marginal pdf of X, Y (iii) $P(X < 3, Y \geq 2)$.   8

5 a) The power spectral density function of a random telegraph signal process is $S(\omega) = \frac{8}{9 + \omega^2}$. Find the corresponding auto correlation function and the power of the process.   7

   b) $\{X(t)\}$ is a random process with mean 2 and auto correlation   8

$R(t_1, t_2) = 5 + 3e^{-0.1|t_1 - t_2|}$. Find the mean, variance and the covariance of the random variables $X(4)$ and $X(6)$.

6 a) The joint PDF of X, Y is $f(x, y) = kxye^{-(x^2+y^2)}, x > 0; y > 0$. Find the value of k, marginal distributions of X, Y and check whether X, Y are independent.   7

b) Let $\{X(t) = A\cos\omega t + B\sin\omega t, t > 0\}$ be a random process where A and B are independent random variables following normal distribution with mean 0 and variance 4. Check whether $\{X(t)\}$ is WSS.   8

PART C
Answer any two questions

7 a) Is the Poisson Process a stationary process? Find the Autocorrelation of the Poisson process.   7

(b) The tpm of a Markov chain with states 1,2,3 is $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}$ and the initial distribution is P(0)= (0.5, 0.3, 0.2). Find $(i) P(X_2) = 2$ $(ii) P(X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 3)$ 8

c) The tpm of a Markov chain is $P = \begin{bmatrix} 0.5 & 0.5 \\ 0.1 & 0.9 \end{bmatrix}$. Find the steady state distribution of the process.   5

8 a) Use Newton's forward difference formula to find y at $x = 1.5$.   7

x	0	1	2	3	4
y	7	10	13	22	43

b) Use Euler's method with $h = 0.025$ to compute the value of $y(0.1)$ for the differential equation $y' = x - y^2$, $y(0) = 1$.   8

c) Use Newton-Raphson method to find $\sqrt{35}$ correct to 4 decimal places.   5

9 a) A man either drives a car or catches a train to go to office every day. He never goes two days in a row by train. But if he drives one day then the next day he is just as likely to drive again as he is to travel by train. On the first day of a week, the man tosses a fair dice and drives to work if he gets a 6. Find the Probability that (i) he takes train on second day (ii) he drives to work on third day (iii) he drives to work in the long run.   10

b) Find the Lagrange's Interpolating polynomial corresponding to given data: $f(0) = 0, f(1) = 1, f(2) = 20$. Hence find $f(1.5)$ 5

c) Evaluate $\int_0^1 e^{-x^2} dx$ using Trapezoidal rule with $h = 0.25$ 5