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Reg No.: \_\_\_\_\_

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY  
SECOND SEMESTER B.TECH DEGREE EXAMINATION(S), DECEMBER 2019

Course Code: MA102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100

Duration: 3 Hours

**PART A**

*Answer all questions, each carries 3 marks*

- 1 Find a general solution of the ordinary differential equation  $y'' + y = 0$  (3)
- 2 Reduce to first order and solve.  $yy'' = 3(y')^2$ . (3)
- 3 Find the particular integral of  $y'' - 4y' - 5y = 4 \cos 2x$ . (3)
- 4 Using a suitable transformation, convert the differential equation  $(x^2 D^2 + xD + 1)y = \log x$  into a linear differential equation with constant coefficients. (3)
- 5 If  $f(x)$  is a periodic function of period  $2\pi$  defined in  $[-\pi, \pi]$ . Write down Euler's Formulas  $a_0, a_n, b_n$  for  $f(x)$ . (3)
- 6 Find the half range Fourier cosine series of the function  $f(x) = x$  in the range  $0 < x < 2$ . (3)
- 7 Find the PDE by eliminating arbitrary function  $\varphi$  from  $xyz = \varphi(x + y + z)$ . (3)
- 8 Solve  $(D + 2D')(D - 3D')^2 z = 0$ . (3)
- 9 Write any three assumptions involved in the derivation of one dimensional wave Equation. (3)
- 10 A tightly stretched string of length  $l$  is fixed at both ends and pulled from its mid point to a height  $h$  and released from rest from this position. Write down the initial and boundary conditions. (3)
- 11 Write all possible solutions of one dimensional heat equation. (3)
- 12 Find the steady state temperature distribution in a rod of length  $l$  if the ends are kept at  $0^\circ\text{C}$  and  $100^\circ\text{C}$ . (3)

**PART B**

*Answer six questions, one full question from each module*

**Module 1**

- 13 a) Solve  $y'' - 2y' + y = 0, y(0) = 1, y'(0) = 2$ . (6)
- b) Find a basis of solutions of the ODE  $(x^2 - x)y'' - xy' + y = 0$ , if  $y_1 = x$  is a (5)

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solution.

OR

- 14 a) Solve the ordinary differential equation  $y''' - 3y'' - 4y' + 6y = 0$ . (6)
- b) Solve the ordinary differential equation  $xy'' + 2y' + xy = 0$ , given that  $y_1 = \frac{\sin x}{x}$  is a solution. (5)

Module II

- 15 a) By the method of variation of parameters, solve  $y'' + 4y = \tan 2x$ . (6)
- b) Solve  $y'' + 2y = x^2 e^{3x}$ . (5)

OR

- 16 a) Solve  $(x+3)^2 y'' - 4(x+3)y' + 6y = 3x$ . (6)
- b) Solve  $x^2 y'' - 4xy' + 6y = x^5$ . (5)

Module III

- 17 a) Find the Fourier series of  $f$  defined by  $f(x) = x - x^2$  in  $(-1, 1)$ . (6)
- b) Expand  $f(x) = c$  in the half range sine-series in  $0 \leq x \leq \pi$ . (5)

OR

- 18 Obtain Fourier series for the function  $f(x) = |\cos x|$ ,  $-\pi \leq x \leq \pi$ . (11)

Module IV

- 19 a) Solve  $r + s + 2t = e^{x+y}$ . (6)
- b) Find the general solution of  $x^2(y-z)p + y^2(z-x)q = (x-y)z^2$ . (5)

OR

- 20 a) Solve  $(D^3 + D^2 D' - D D'^2 - D'^3)z = e^x \cos 2y$ . (6)
- b) Solve  $(D^2 + 3DD' + 2D'^2)z = x^2 y^2$ . (5)

Module V

- 21 A uniform elastic string of length 60 cm is subjected to a constant tension of 2 Kg. If the ends are fixed, the initial displacement  $u(x, 0) = 60x - x^2$ ,  $0 < x < 60$  and the initial velocity is zero, find the displacement function  $u(x, t)$ .

OR

- 22 Find the deflection of the vibrating string which is fixed at the ends  $x = 0$  and  $x = 2$  and the motion is started by displacing the string into the form  $\sin^3\left(\frac{\pi x}{2}\right)$  and released it with zero initial velocity at  $t = 0$ . (10)

**Module VI**

- 23 Find the temperature distribution in a rod of length  $2m$  whose endpoints are maintained at temperature zero and initial temperature is  $f(x) = 100(2x - x^2)$ . (10)

**OR**

- 24 A rod of length 30cm has its ends A and B kept at  $20^\circ C$  and  $80^\circ C$  respectively until steady state temperature prevails. Suddenly the temperature at A is raised to  $60^\circ C$  and the end B is decreased to  $40^\circ C$ . Find the temperature distribution in the rod at time  $t$ . (10)

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