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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION(S), DECEMBER 2019

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks. Marks

1 a) Find the sum of the series $\sum_{k=1}^{\infty}\frac{2}{3^{(k+1)}}$ (2)

b) Determine whether the alternating series $\sum_{k=2}^{\infty}(-1)^k\frac{k}{k-1}$ converges. (3)

2 a) Find the slope of the function $f(x,y) = x\cos(xy) + y\sin(xy)$ at $(\pi,1)$ along the x- direction. (2)

b) If $z= f(x^2-y^2)$, show that

$$y\frac{\partial z}{\partial x}+x\frac{\partial z}{\partial y}=0$$ (3)

3 a) Find $\lim_{t\to 0} \boldsymbol{r}(t)$, where $\boldsymbol{r}(t) = \langle 1+t^3, te^{-t}, \frac{\sin t}{t}\rangle$ (2)

b) Find the directional derivative of $f(x,y) = e^x\cos y$ at $P(0,\pi/4)$ in the direction of negative Y-axis (3)

4 a) Evaluate $\int_0^1\int_0^1\int_0^1 e^{x+y+z}\,dx\,dy\,dz$ (2)

b) Evaluate $\iint_R \left(x^2+y^2\right)dx\,dy$ where R is the region taken over the first quadrant for which $x+y\le 1$. (3)

5 a) Find the divergence of the vector field $F(x,y,z) = x^2y\,i + 2y^3z\,j + 3z\,k$ (2)

b) Evaluate $\int_c x^2dy + y^2dx$ where C is the path $y = x$ from (0,0) to(1,1) (3)

6 a) Determine the source and sink of the vector field $F(x,y,z) = 2(x^3-2x)i + 2(y^3-2y)j + 2(z^3-2z)k$ (2)

b) If S is any closed surface enclosing a volume V and if $A = axi + byj + czk$ prove that $\iint A.nds = (a+b+c)V$ (3)

PART B

Module 1

Answer any two questions, each carries 5 marks.

7 Test for convergence of the series $\sum_{k=0}^{\infty}\frac{(k!)^2}{(2k)!}$. (5)

8 Find the radius of convergence of $\sum_{k=0}^{\infty}\frac{(2x-1)^k}{3^{2k}}$. (5)

9 Expand $f(x) = \sin\pi x$ into a Taylors series about $x = \frac{1}{2}$,up to third derivative. (5)

Module 1I

Answer any two questions, each carries 5 marks.

10 If $u = f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$ find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. (5)

11 Find the local linear approximation L(x ,y) of f(x ,y)=$\frac{1}{\sqrt{x^2+y^2}}$ at the point P(4,3).Compare the error in the approximation to f by L at the point Q(3.92,3.01) with the distance between P and Q. (5)

12 Locate all relative extrema and saddle point for the function $f(x,y) = x^3 + y^3 - 6xy + 20$. (5)

Module 1II

Answer any two questions, each carries 5 marks.

13 Find the equation of the unit tangent and unit normal to the curve $x = e^t\cos t, y = e^t\sin t, z = e^t$; at $t = 0$. (5)

14 A particle moves along the curve $r(t) = \left(\frac{1}{t}\right)i + t^2j + t^3k$, where t denotes time. Find (5)

1) The scalar tangential and normal components of acceleration at time $t = 1$.

2) The vector tangential and normal component of acceleration at time $t = 1$

15 Find the equation of the tangent plane and the parametric equations of the normal line to the surface $z = 4x^3y^2 + 2y - 2$ at (1,-2,10). (5)

Module 1V

Answer any two questions, each carries 5 marks.

16 Use double integral to find the area of the plane enclosed by $y^2 = 4x$ and $x^2 = 4y$ (5)

17 Change the order of integration to evaluate $\int_0^1\int_{x^2}^{2-x} xy\,dy\,dx$ (5)

18 Use triple integral to find the volume of the solid with in the cylinder $x^2+y^2=4$ and between the planes $z=0$ and $y+z=3$. (5)

Module V

Answer any three questions, each carries 5 marks.

19 If $\bar{r}=x\bar{\imath}+y\bar{\jmath}+z\bar{k}$ and $r=|\bar{r}|$, prove that $\nabla^2 r^n = n(n+1)\,r^{n-2}$ (5)

20 Evaluate $\int_C (3x^2+y^2)\,dx + 2xydy$ along the curve $C: x=\cos t, y=\sin t, 0\le t\le\frac{\pi}{2}$ (5)

21 Find the scalar potential of $\vec{F}=(2xy+z^3)\vec{\imath}+x^2\vec{\jmath}+3xz^2\vec{k}$ (5)

22 Find the work done by $F(x,y)=(x+y)i+xy\,j-z^2\,k$ along the line segments from $(0,0,0)$ to $(1,3,1)$ to $(2,-1,5)$ (5)

23 Show that $\int_{(0,0)}^{(1,\frac{\pi}{2})} e^x\sin y\,dx + e^x\cos y\,dy$ is independent of path. Hence evaluate $\int_{(0,0)}^{(1,\frac{\pi}{2})} e^x\sin y\,dx + e^x\cos y\,dy$ (5)

Module VI

Answer any three questions, each carries 5 marks.

24 Evaluate using Green's theorem in the plane $\int_c (x^2dx - xydy)$ where C is the boundary of the square formed by $x=0, y=0, x=a, y=a$ (5)

25 Evaluate the surface integral $\iint_\sigma f(x,y,z)\,ds$ where $f(x,y,z)=x+y$, σ is the portion of the surface $z=6-2x-4y$ in the first octant. (5)

26 Using divergence theorem find the flux across the surface σ which is the surface of the tetrahedron in the first octant bounded by $x+y+z=1$ and the coordinate planes, $\bar{F}=(x^2+y)\bar{\imath}+xy\bar{\jmath}-(2xz+y)\bar{k}$ (5)

27 Evaluate $\int_c (e^x dx + 2ydy - dz)$ where C is the curve $x^2+y^2=4, z=2$ using Stoke's theorem (5)

28 Evaluate the surface integral $\iint_\sigma f(x,y,z)\,ds$ where $f(x,y,z)=x^2+y^2$, σ is the surface of the sphere $x^2+y^2+z^2=a^2$ (5)

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