APJ ABDULKALAM TECHNOLOGICAL UNIVERSITY
08 PALAKKAD CLUSTER

Q. P. Code : CS-1D-19-1 **(Pages: 3)** **Name:** ....

Reg. No:..............................

FIRST SEMESTER M.TECH. DEGREE EXAMINATION DEC 2019

Branch: Computer Science and Engineering Specialization: Computer Science and Engineering

08 CS 6041 MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

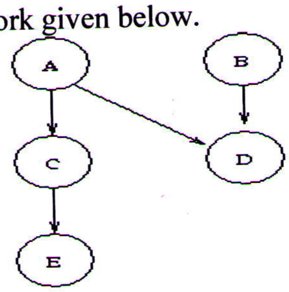
Time:3 hours **Max.marks: 60**

Answer all six questions.

Modules 1 to 6: Part ‘a’ of each question is compulsory and answer either part ‘b’ or part ‘c’ of each question.

Q. No	Module 1	Marks
1a	Define Characteristic equation of a matrix. What is Eigen value and Eigen Vector? What are its properties?	3
b	Find the SVD of the matrix $A = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$	6
c	Find the LU decomposition of the matrix $A = \begin{bmatrix} 1 & 4 & -3 \\ -2 & 8 & 5 \\ 3 & 4 & 7 \end{bmatrix}$	6

Q. No	Module 2	Marks
2a	State Baye’s Theorem. Define Bayesian Network?	3
b	Consider the Bayesian network given below.	6



It follows the probabilities

Prob(A=T) = 0.3
Prob(B=T) = 0.6
Prob(C=T|A=T) = 0.8
Prob(C=T|A=F) = 0.4

Prob(D=T|A=T,B=T) = 0.7
Prob(D=T|A=T,B=F) = 0.8
Prob(D=T|A=F,B=T) = 0.1
Prob(D=T|A=F,B=F) = 0.2
Prob(E=T|C=T) = 0.7
Prob(E=T|C=F) = 0.2

Compute

i. Prob(D=T)
ii. Prob(A=T| C=T)
iii. Prob(A=T,D=T| B=F)

c The probability of A_1, A_2 and A_3 becoming managers are 4/9, 2/9 and 1/3 respectively. The probabilities that the bonus scheme will be introduced if A_1, A_2 and A_3 become managers are 1/10, 1/2 and 4/5 respectively. If the bonus scheme is introduced, what is the probability that each one becomes the manager? 6

Q. No	Module 3	Marks
3a	State Chapman Kolmogorov Theorem for Homogenous Markov Chain? Define its properties?	3
b i.	There are three super markets A, B, C opened in a city. 80% of the adults in the city go to Super market A the rest go to supermarket B. 40% of the village people goes to supermarket B and rest equally chooses A and C. 70% of the people in the neighbouring city visits C, 20% visits A and the remaining visits B. Draw the transition diagram and the transition probability matrix.	2
ii.	If the transition probability matrix $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}$ and the initial probability $P(0) = [0.3 \quad 0.5 \quad 0.2]$ a. $P[X_2 = 2]$ b. $P[X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 3]$	4
c	With examples explain the different states of Markov Chain	6

Q. No	Module 4	Marks
4a.	State Birth-Death process? What is the steady–state solution to a Birth-Death process?	3
b	A mobile shop sells on an average of 5 mobiles per week. Using Poisson process find the probability of selling , a. Some mobile phones b. 4 or more phones but less than 6 c. Assume there are 6 working days per week, what is the probability that in a given day the shop will sell 1 mobile phone	6

Q. No		Marks
c	Consider CTMC with transition probability $P = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ with holding time parameters $\lambda_1= 2$ $\lambda_2= 1$ and $\lambda_3=3$. Find the limiting distribution and the generator matrix	6

Q. No	Module 5	Marks
5a	What are the basic characteristics of a Queuing Model? Define the Kendall's Notation of a Queuing System? Define the Little's formula for M/M/1: infinity/FIFO model	4
b	An ice-cream parlour having two counters for serving. The customer arrive in a Poisson rate of 10 per hour. The service time for each customer is exponentially distributed with mean of 10 minutes. Find 1. Probability of no customers in the system 2. Probability that a customer has to wait for service 3. Average number of customers in the queue 4. Average number of customers in the system 5. Expected waiting time of customers in the queue 6. Expected time a customer spends in the system	8
c	A car service station has two service bays which can service two cars at a time. The station can accommodate 6 cars in the queue waiting for service and others need to leave the station without servicing. The average arrival rate is 20 per hour and spend an average of 4minutes in the station. The arrival process is Poisson and the service time is exponential random variable. Find, 1. P_o, P_1 and P_8 2. The effective arrival rate at car service station 3. The expected number of cars in the queue 4. The expected number of cars at the service station 5. Expected waiting time of cars in the queue 6. Expected time a car spends at the service station	8

Q. No	Module 6	Marks
6a	**i.** Write down the Pollaczek Khinchine formula for L_s , L_q, W_s and W_q in the M/G/1 queuing system	2
	ii. With example explain Queue networks	2
b	Derive Pollaczek Khinchine formula for the average number of customers in the M/G/1 queuing system	8
c	Customer arrive at a hotel according to Poisson distribution with an average arrival rate of 5 per hour. It is estimated that the service time follows a random distribution with mean service time of 10 minutes and standard deviation equal to 6 minutes.Find L_s , L_q, W_s and W_s.	8