Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION(R&S), DECEMBER 2019

Course Code: MA201

Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer any two full questions, each carries 15 marks Marks

1 a) Check whether the function $f(z) = \begin{cases} Re\left(\frac{z^2}{|z|}\right), & z \neq 0 \\ 0, & z = 0 \end{cases}$ is continuous at $z = 0$. (7)

b) Show that if $f(z) = u(x,y) + iv(x,y)$ is analytic, then $u(x,y)$ and $v(x,y)$ satisfy Cauchy- Riemann equations. (8)

2 a) Determine the region in the w −plane into which the triangular region bounded by $x = 1$, $y = 1$ and $x + y = 1$ is mapped by $w = z^2$. (7)

b) Find the linear fractional transformation that maps $(-2, 0, 2)$ onto $\left(\infty, \frac{1}{4}, \frac{3}{8}\right)$. Under this transformation what is the image of the x − axis. (8)

3 a) Find the real part of an analytic function whose imaginary part is $v = e^{-x}(x\,cosy + y\,siny)$. Also find the corresponding analytic function. (7)

b) Prove that $w = \frac{z}{1-z}$ maps the upper half plane $y > 0$ into the upper half plane of w −plane. What is the image of $|z| = 1$ under this mapping? (8)

PART B

Answer any two full questions, each carries 15 marks

4 a) Use Cauchy's Integral formula to evaluate $\oint_C \frac{z^2+1}{z^2-1} dz$ counter clock wise around (7)

$(i)|z-1| = 1$ $(ii)\ |z+1| = 1$

b) Find the Laurent's series of $\frac{1}{(z-1)(z-2)}$ in (8)

$(i)\ 1 < |z| < 2$ $(ii)\ |z| > 2$ $(ii)\ 0 < |z-1| < 1$

5 a) Use Cauchy's Residue theorem to evaluate $\oint_C \left(\frac{Ze^{\pi z}}{z^4-16}\right) dz$, where C is the ellipse $9x^2 + y^2 = 9$. (7)

b) Evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2}-\cos\theta}$ using contour integration. (8)

6 a) Evaluate $\int (Re\ z)\ dz$ along the real axis from 0 to 1 and then along a straight line parallel to imaginary axis from 1 to $1 + 2i$. (7)

b) Evaluate $\int_{-\infty}^{\infty} \frac{1}{(x^2+1)^2} dx$ using contour integration. (8)

PART C

Answer any two full questions, each carries 20 marks

7 a) Solve the system of equations using Gauss Elimination method:
$y + z - 2w = 0, \quad 2x - 3y - 3z + 6w = 2, \quad 4x + y + z - 2w = 4$ (8)

b) If the matrix $\begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & -2 & a & b \end{bmatrix}$ is of rank **2**, find the values of a, b. (6)

c) Check whether the vectors [1, 2, 1], [2, 1, 4], [4, 5, 6], [1, 8, −3] are linearly dependent in R^3. (6)

8 a) Diagonalise the symmetric matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ (8)

b) If one eigen values of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ is **5**, find the other eigen values without finding the characteristic equation. What are the eigen values of A^2 and A^{-1}. (6)

c) Reduce the quadratic form $q = 3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form. Examine the definiteness. (6)

9 a) Find a matrix B which transform $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ in to the diagonal form. (10)

b) Find a basis and dimension for the row space, column space and null space for the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 2 & 5 \\ -2 & -5 & 1 & -1 & -8 \\ 0 & -3 & 3 & 4 & 1 \\ 3 & 6 & 0 & -7 & 2 \end{bmatrix}$ (10)

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