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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIFTH SEMESTER B.TECH DEGREE EXAMINATION(R&S), DECEMBER 2019

Course Code: CS309

Course Name: GRAPH THEORY AND COMBINATORICS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries3 marks. Marks

1 Define the terms a) Walk b) Path and c) Circuit with an example. (3)

2 Prove that the no of vertices of odd degree in a graph is always even (3)

3 Draw a graph that has a Hamiltonian path but does not have a Hamiltonian circuit. (3)

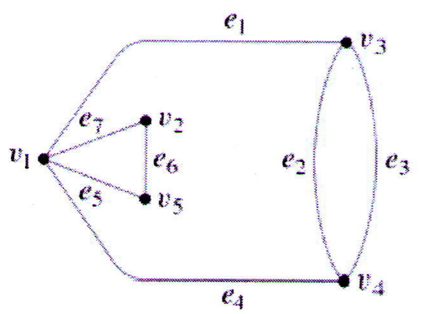
4 Differentiate between Symmetric and Asymmetric digraphs with examples and draw a complete symmetric digraph of four vertices. (3)

PART B

Answer any two full questions, each carries9 marks.

5 a) Prove that a simple graph with n vertices and k components can have at most (n-k)(n-k+1)/2 edges (4)

b) Define Isomorphism of graphs. Check whether the two graphs are isomorphic or not (5)



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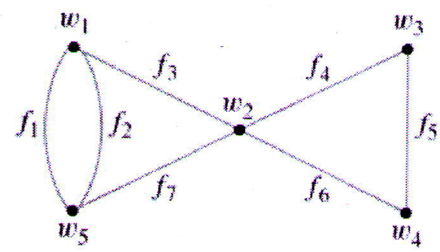
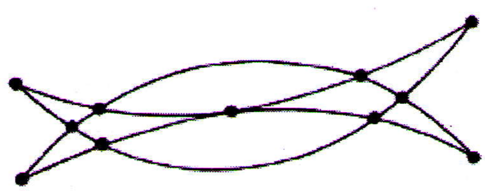


G'

6 a) Define Euler graph. Check whether the graph is an euler graph or not. If yes, give the Euler line and justify your answer. (5)



b) Prove that a connected graph G is an Euler graph if and only if all vertices of G are of (4) even degree.

7 a) State travelling salesman problem and how TSP solution is related to Hamiltonian circuits. (5)

b) State and Prove Dirac's Theorem for Hamiltonicity. (4)

PART C

Answer all questions, each carries3 marks.

8 Prove that the distance between the vertices of a connected graph is a metric (3)

9 List down any two properties of a tree and also prove the following theorem: A graph is a tree if and only if it is minimally connected. (3)

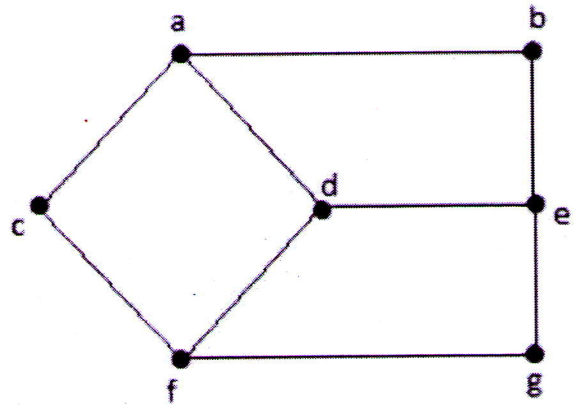
10 Define the terms vertex connectivity and edge connectivity with examples. (3)

11 Give the different representations of a planar graph. (3)

PART D

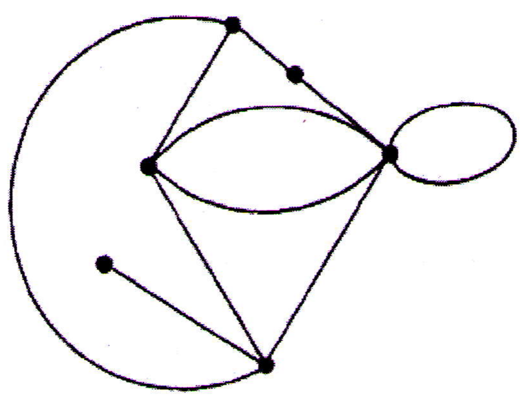
Answer any two full questions, each carries9 marks.

12 a) Find the eccentricity of all vertices in the graph G given below and also mark the center, radius and diameter of G (6)



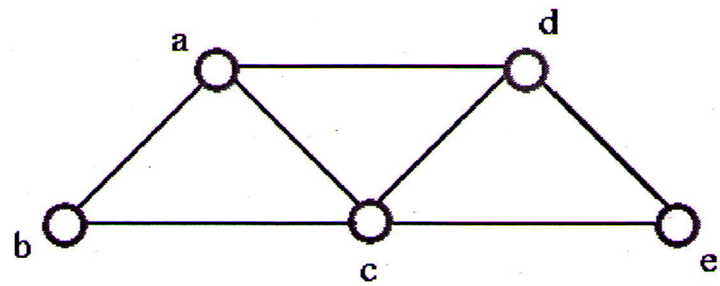
b) State and prove Cayley's theorem (3)

13 a) Find the Geometrical dual (G*) of the graph G given below (5)



b) List out the properties stating the relationship between the graph G and its dual G* (4)

14 a) Consider the graph G and any one of its spanning tree T. Find all fundamental circuits and fundamental cut sets with respect to the spanning tree T. (6)



b) Prove that "Every cut set in a connected graph G must contain atleast one branch of every spanning tree of G". (3)

PART E

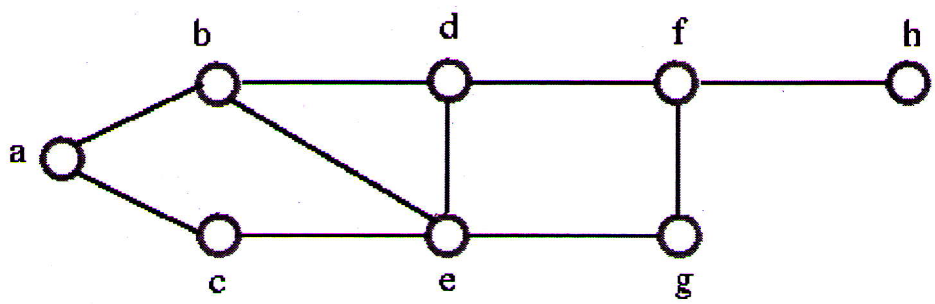
Answer any four full questions, each carries10 marks.

15 a) Define Adjacency Matrix X(G) of a graph. Determine the properties of adjacency matrix. (6)

b) Draw the graph represented by the following adjacency matrix. (4)

$$X(G) = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

16 a) Obtain a cut set matrix for the following graph (6)



b) Define path matrix. Determine the properties of a path matrix. (4)

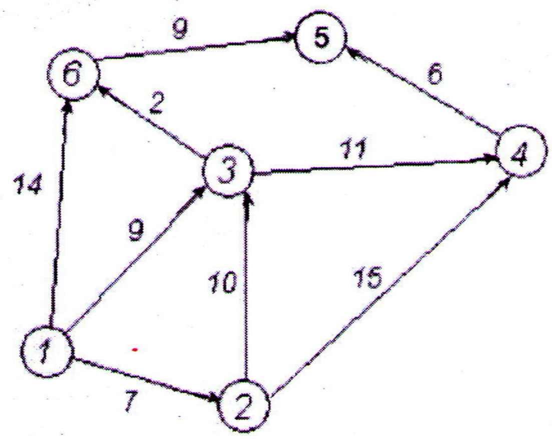
17 a) Explain edge listing and successor listing methods used in computer representation of graphs (4)

b) Draw the flow chart to determine connectedness and components of a graph (6)

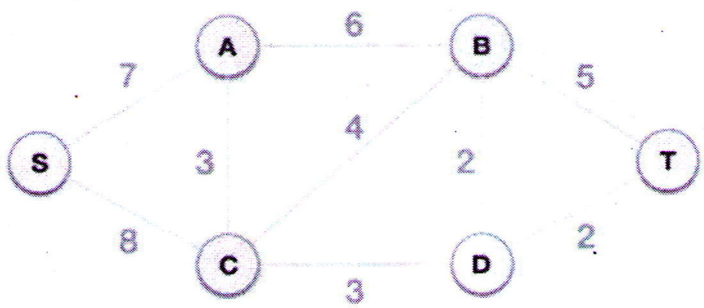
18 Draw a flowchart indicating all the five conditions to find the spanning tree /spanning (10)

forest. Apply it to find the spanning tree /spanning forest for any graph of your choice.

19 Write Dijkstra's Shortest path algorithm and apply this algorithm to find the shortest path (10)



20 Write Kruskal's algorithm to find the minimum spanning tree of a graph G .Apply it to find the MST for the graph given below (10)



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