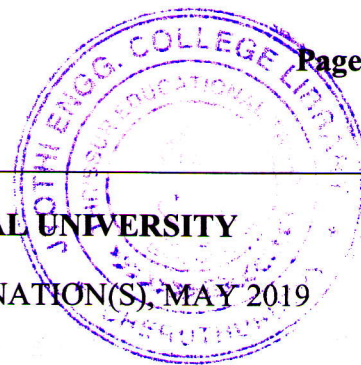


Reg No.: _____

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION(S), MAY 2019

Course Code: MA201

Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer any two full questions, each carries 15 marks

- | | Marks |
|---|-------|
| 1 a) Prove that the function $\sin z$ is analytic and find its derivative. | (7) |
| b) Under the transformation $w = \frac{1}{z}$, find the image of $ z - 2i = 2$ | (8) |
| 2 a) Find the analytic function whose imaginary part is | (7) |
| $v(x, y) = \log(x^2 + y^2) + x - 2y$. | |
| b) Under the transformation $w = z^2$, find the image of the triangular region bounded by $x = 1$, $y = 1$ and $x + y = 1$. | (8) |
| 3 a) Show that $f(z) = \begin{cases} \frac{z \operatorname{Re}(z)}{ z }, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not differentiable at $z = 0$ | (7) |
| b) Find the bilinear transformation that maps the points $-1, i, -1$ onto $i, 0, -i$. | (8) |

PART B

Answer any two full questions, each carries 15 marks

- | | |
|---|-----|
| 4 a) Using Cauchy's integral formula, evaluate $\int_C \frac{e^z}{(z^2+4)(z-1)^2} dz$, where C is the circle $ z - 1 = 2$. | (7) |
| b) Evaluate $\int_0^{2+i} (\bar{z})^2 dz$ along | (8) |
| (i) the real axis to 2 and then vertically to $2 + i$. | |
| (ii) the line $2y = x$ | |
| 5 a) Find all singular points and residues of the functions | (7) |

$$(a) f(z) = \frac{z - \sin z}{z^2} \quad (b) f(z) = \tan z$$

$$b) \text{ Evaluate } \int_0^{2\pi} \frac{1}{5 - 3\sin\theta} d\theta . \quad (8)$$

$$6 \ a) \text{ Evaluate } \int_C \log z dz \text{ where } C \text{ is the circle } |z| = 1 \quad (7)$$

$$b) \text{ Evaluate } \int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx \quad (8)$$

PART C

Answer any two full questions, each carries 20 marks

$$7 \ a) \text{ Find the rank of the matrix } \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix} \quad (8)$$

$$b) \text{ Find the values of } a \text{ and } b \text{ for which the system of linear equations} \quad (7)$$

$$x + 2y + 3z = 6, x + 3y + 5z = 9, 2x + 5y + az = b \text{ has (i) no solution}$$

(ii) a unique solution (iii) infinitely many solutions

$$c) \text{ Show that the vectors } [3 \ 4 \ 0 \ 1], [2 \ -1 \ 3 \ 5] \text{ and } [1 \ 6 \ -8 \ -2] \quad (5)$$

are linearly independent in \mathbb{R}^4 .

$$8 \ a) \text{ Solve the system of equations by Gauss Elimination Method:} \quad (8)$$

$$3x + 3y + 2z = 1, \quad x + 2y = 4, \quad 10y + 3z = -2, \quad 2x - 3y - z = 5$$

$$b) \text{ Find the nature, index, rank and signature of the quadratic form} \quad (6)$$

$$x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 - 2x_1x_3 + 2x_2x_3$$

$$c) \text{ Find the Eigen values and Eigen vectors of } \begin{bmatrix} 4 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix} \quad (6)$$

$$9 \ a) \text{ Diagonalize the matrix } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \quad (8)$$

$$b) \text{ Define symmetric and skew symmetric matrices. Show that any real square} \quad (6)$$

matrix can be written as the sum of a symmetric and a skew symmetric matrix.

$$c) \text{ What type of conic section is represented by the quadratic form} \quad (6)$$

$$3x^2 + 22xy + 3y^2 = 0 \text{ by reducing it into canonical form.}$$
