E | **E5840** | **Pages: 3**

Reg No.:_______________ Name:_______________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIFTH SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: CS309

Course Name: GRAPH THEORY AND COMBINATORICS

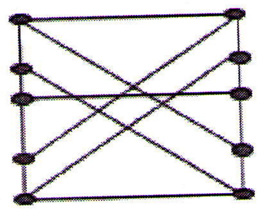
Max. Marks: 100 Duration: 3 Hours

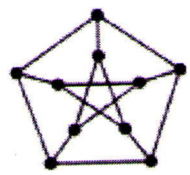
PART A

Answer all questions, each carries 3 marks

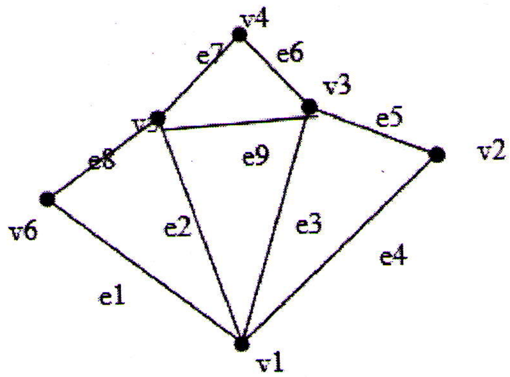
Marks

1 Define isomorphism between two graphs. Are the following graphs are isomorphic to each other? Justify your answer. (3)





2 For the following graph, find the shortest path between fromv1 to v4. Also find a Euler circuit. (3)



3 Define the following with example. (3)

i) Isomorphic digraph ii) Complete symmetric digraph

4 Define Hamiltonian graph.Find an example of a non-Hamiltonian graph with a Hamiltonian path. (3)

PART B

Answer any two full questions, each carries 9 marks

5 a) For a Eulerian graph G, prove the following properties. (6)

i) The degree of each vertex of G is even. ii) G is an edge-disjoint union of cycles.

b) Discuss the Konigsberg Bridge problem.Is there any solution to the problem? Justify your answer. (3)

6 a) Prove that a simple graph with n vertices must be connected, if it has more than (n-1)(n-2)/2 edges. (6)

b) 19 students in a nursery school play a game each day, where they hold hands to form a circle. For how many days can they do this, with no students holding hands with the same playmates more than once? Substantiate your answer with graph theoretic concepts. (3)

7 a) Prove that the number of odd degree vertices in a graph is always even. (4)

b) Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group. (5)

PART C

Answer all questions, each carries 3 marks

8 Discuss the dual of a subgraph with example. (3)

9 Write notes on the fundamental circuit. (3)

10 Prove that in a tree T(V,E),|V|=|E|+1. (3)

11 Define spanning tree with example. (3)

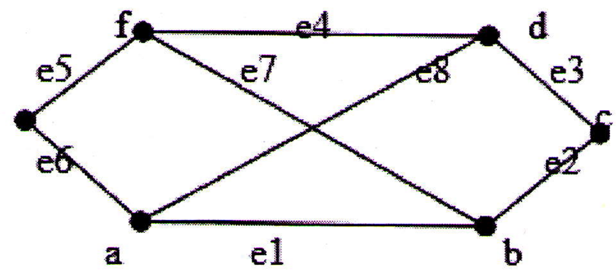
PART D

Answer any two full questions, each carries 9 marks

12 Prove that the ring sum of any two cut-sets in a graph is either a third cut-set or an edge-disjoint union of cut-sets. (9)

13 a) Prove that a connected planar graph with n vertices and e edges has e-n+2 regions. (4)

b) Consider the following graph G and any one of its spanning trees, T.List all fundamental circuits and fundamental cut-sets with respect to T. (5)



14 a) Show that the distance between vertices of a connected graph is a metric. (6)

b) Discuss the center of a tree with suitable example. (3)

PART E

Answer any four full questions, each carries 10 marks

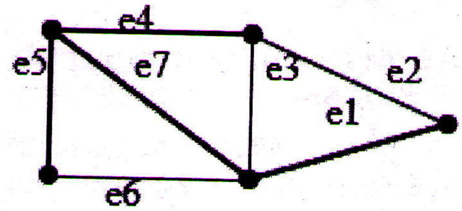
15 a) Define the adjacency matrix X(G) of a graph. Let X(G) be adjacency matrix of a simple graph G, then prove that ij^{th} entry in X^r is the number of different edge sequences of r edges between vertices $v_i$and V_j. (6)

b) Draw the adjacency graph for the following adjacency matrix. (4)

$$X(G) = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

16 a) Define the circuit-matrix B(G) of a connected graph G with n vertices and e edges. Prove that the rank of B(G) is e-n+1. (6)

b) Write the fundamental circuit matrix with respect to the spanning tree shown in heavy lines for the following graph. (4)



17 Discuss an algorithm for finding the shortest path from a specified vertex to another specified vertex. Illustrate with example. (10)

18 Discuss an algorithm for finding the connected components of a graph G with suitable example. (10)

19 Discuss an algorithm to find the minimum spanning tree of a graph G with example. (10)

20 a) Define the incidence matrix of a graph G.Prove that the rank of an incidence matrix of a connected graph with n vertices is n-1. (6)

b) Draw the graph represented by the following incidence matrix. (4)

$$X(G) = \begin{pmatrix} 1&0&0&0&0&1 \\ 0&1&1&0&1&0 \\ 1&0&0&1&0&0 \\ 0&1&0&1&0&0 \\ 0&0&1&0&0&1 \end{pmatrix}$$

****