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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks.

Marks

1 a) Determine whether the series $\sum_{k=0}^{\infty}\frac{5}{4^k}$ converges. If so, find the sum (2)

b) Examine the convergence of $\sum\left(\frac{k}{k+1}\right)^{k^2}$ (3)

2 a) Find the slope of the surface $z = x\,e^{-y} + 5y$ in the y direction at the point (4, 0) (2)

b) Show the function $f(x,y) = e^x \sin y + e^y \cos x$ satisfies the Laplace's equation $f_{xx} + f_{yy} = 0$ (3)

3 a) Find the directional derivative of $f(x,y,z) = x^3z - yx^2 + z^2$ at P (2, -1, 1) in the direction of $3\,\vec{i} - \boldsymbol{j} + 2\mathbf{k}$ (2)

b) Find the unit tangent vector and unit normal vector to the curve (3)

$\boldsymbol{r}(t) = 4\cos t\,\boldsymbol{i} + 4\sin t\,\boldsymbol{j} + t\,\boldsymbol{k}$ at $t = \frac{\pi}{2}$

4 a) Using double integration, evaluate the area enclosed by the lines (2)

$$x = 0, \quad y = 0, \frac{x}{a}+\frac{y}{b} = 1$$

b) Evaluate $\int_{-1}^{2}\int_{0}^{2}\int_{0}^{1}(x^2 + y^2 + z^2)\,dx\,dy\,dz$ (3)

5 a) If $\boldsymbol{F}(x,y,z) = x^2\boldsymbol{i} - 3\boldsymbol{j} + yz^2\boldsymbol{k}$ find div $\boldsymbol{F}$ (2)

b) Find the work done by the force field $\boldsymbol{F} = xy\,\boldsymbol{i} + yz\,\boldsymbol{j} + zx\,\boldsymbol{k}$ on a particle that moves along the curve C: $x = t, y = t^2, z = t^3, 0 \le t \le 1$ (3)

6 a) Use Green's theorem to evaluate $\int_c (xdy - ydx)$, where c is the circle $x^2 + y^2 = a^2$ (2)

b) If S is any closed surface enclosing a volume V and $\boldsymbol{F} = x\boldsymbol{i} + 2y\boldsymbol{j} + 3z\boldsymbol{k}$ show that $\iint_S \mathrm{F.n}\,ds = 6V$ (3)

PART B

Module I

Answer any two questions, each carries 5 marks.

7 Determine whether the alternating series $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k+7}{k(k+4)}$ is absolutely convergent (5)

8 Find the Taylor series expansion of $f(x) = \frac{1}{x+2}$ about $x = 1$ (5)

9 Find the interval of convergence and radius of convergence of $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{(x+1)^k}{k}$ (5)

Module II

Answer any two questions, each carries 5 marks.

10 Find the local linear approximation L to the function $f(x,y,z) = xyz$ at the point P (1,2,3). Also compare the error in approximating f by L at the point Q (1.001, 2.002, 3.003) with the distance PQ. (5)

11 Locate all relative extrema and saddle points of $f(x,y) = 2xy - x^3 - y^2$ (5)

12 If $u = f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ (5)

Module III

Answer any two questions, each carries 5 marks.

13 Write the parametric equations of the tangent line to the graph of $r(t) = ln\, t\, i + e^{-t}\boldsymbol{j} + t^4\boldsymbol{k}$ at $t = 2$ (5)

14 A particle moves along the curve $\boldsymbol{r} = (t^3 - 4t)\,\boldsymbol{i} + (t^2 + 4t)\,j + (8t^2 - 3t^3)\boldsymbol{k}$ where t denotes time. Find (5)
(i) the scalar tangential and normal components of acceleration at time t = 2
(ii) the vector tangential and normal components of acceleration at time t = 2

15 Find the equation to the tangent plane and parametric equations of the normal line to the ellipsoid $x^2 + y^2 + 4z^2 = 12$ a t the point (2, 2, 1) (5)

Module IV

Answer any two questions, each carries 5 marks.

16 Reverse the order of integration and evaluate $\int_0^1\int_x^1 \frac{x}{x^2+y^2}dydx$ (5)

17 If R is the region bounded by the parabolas $y = x^2$ and $y^2 = x$ in the first quadrant, evaluate $\iint_R (x+y)dA$ (5)

18 Use triple integral to find the volume of the solid bounded by the surface $y = x^2$ and the planes $y + z = 4, z = 0$. (5)

Module V

Answer any three questions, each carries 5 marks.

19 If $\boldsymbol{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$ and $r = ||\boldsymbol{r}||$, show that $\nabla log\, r = \frac{\boldsymbol{r}}{r^2}$ (5)

20 Examine whether $\boldsymbol{F} = (x^2 - yz)\boldsymbol{i} + (y^2 - zx)\boldsymbol{j} + (z^2 - xy)\boldsymbol{k}$ is a conservative field. If so, find the potential function (5)

21 Show that $\nabla^2 f(r) = 2\frac{f'(r)}{r} + f''(r)$, where $\boldsymbol{r} = x\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$, $r = ||\,\boldsymbol{r}||$ (5)

22 Compute the line integral $\int_c (y^2dx - x^2dy)$ along the triangle whose vertices are (1, 0), (0,1) and (-1, 0) (5)

23 Show that the line integral $\int_c (y\sin x dx - \cos x dy)$ is independent of the path and hence evaluate it from (0, 1) and (π, - 1) (5)

Module VI

Answer any three questions, each carries 5 marks.

24 Using Green's theorem, find the work done by the force field $\vec{f}(x,y) = (e^x - y^3)\vec{i} + (\cos y + x^3)\vec{j}$ on a particle that travels once around the unit circle $x^2 + y^2 = 1$ in the counter clockwise direction. (5)

25 Using Green's theorem evaluate $\int_c (xy + y^2)dx + x^2dy$, where c is the boundary of the area common to the curve $y = x^2$ and $y = x$ (5)

26 Evaluate the surface integral $\iint_S xzds$ **,** where S is the part of the plane $x + y + z = 1$ that lies in the first octant (5)

27 Using divergence theorem, evaluate $\iint_S \mathsf{F.n}\,ds$ where $\boldsymbol{F} = (x^2 + y)\,\boldsymbol{i} + z^2\,\boldsymbol{j} + (e^y - z)\,\boldsymbol{k}$ and S is the surface of the rectangular solid bounded by the co ordinate planes and the planes x = 3, y = 1, z = 3 (5)

28 Apply Stokes's theorem to evaluate $\int_c F.dr$, where $\boldsymbol{F} = (x^2 - y^2)\boldsymbol{i} + 2xy\boldsymbol{j}$ and c is the rectangle in the xy plane bounded by the lines $x = 0, y = 0, x = a$ and $y = b$ (5)

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