| APJ ABDUL KALAM TECHNOLOGICAL UNITERSITY FOURTH SEMESTER B.TECH DEGREE EXAMINATION APHI- FOURTH SEMESTER B.TECH DEGREE EXAMINATION APHI- FOURSE Name: PROBABILITY, RANDOM PROCESSES AND NUMERAL METHODS (AE, EC)Max. Marks: 100Duration: 3 Hours (Normal distribution table is allowed in the examination hall) PART A Answer any two full questions, each carries 15 marksMarks1a) A random variable X has the following probability distribution:(7) \overline{x} -2 -1 0 1 2 3 Find: i) The value of kii) Evaluate $P(X < 2)$ and $P(-2 < X < 2)$ iii) Evaluate the mean of Xb)The probability that a component is acceptable is 0.93 . Ten components are picked at random. What is the probability that: i) At least nine are acceptable(8)2a)Suppose that the length of a phone call in minutes is an exponential random variable telephone booth, find the probability that you will have to wait: i) More than 10 minutesii) Between 10 and 20 minutes.b)For a normally distributed population, 7% of items have their values less than 35 and 89% have their values less than 63. Find the mean and standard deviation of the distribution.(8)3a)Fit a binomial distribution to the following data and calculate the theoretical frequencies.(8)b)The time between breakdowns of a particular machine follows an exponential(7) | A Reg N | o.: | | | | E4802 | 2 | Name: | CI-HI C | STOT | ATION STREET | 2 | |
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| 11 + 12 + 13 $11 + 12 + 13$ $12 +$ | b | | | | | | | | | | | | |
| distribution, with a mean of 17 days. Calculate the probability that a machine breaks | | | | | of 1 / da | ys. Cai | culate | the pro | oadiniy | inat a m | |) | |
| down in a 15 day period. PART B | | down in | a 15 day pe | erioa. | | DA | рт р | | | | | | |
| Answer any two full questions, each carries 15 marks | | | Ans | wer an | v two fu | | | each car | rries 15 | marks | | | |
| 4 a) The joint PDF of two continuous random variables X and Y is given by (7) | 4 a |) Th | | | | | | | | | given by | (7) | |
| $f(x,y) = \begin{cases} kxy & 0 < x < 4, \ 1 < y < 5\\ 0 & otherwise \end{cases}.$ | | | - | | | | | | | | | | |
| | | | | | | | | | | | | | |
| Find: i) k ii) The marginal distributions of X and Y iii) Check whether X and y are independent. | | Find: | , | 1 | | - | | | | ľ | | | |

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b) A distribution with unknown mean μ has variance equal to 1.5. Use Central Limit (8) Theorem to find how large a sample should be taken from the distribution in order that the probability will be at least 0.95 that the sample mean will be within 0.5 of the population mean.

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- 5 a) The autocorrelation function for a stationary process X(t) is given by R_{XX}(τ) = (7) 9 + 2e^{-|τ|}. Find the mean value of the random variable Y = ∫_{τ=0}² X(t)dt and the variance of X(t).
 b) A random process X(t) is defined by X(t) = Y(t) cos(ωt + θ) Where Y(t) is a (8)
 - WSS process, ω is a constant and θ is a random variable which is uniformly distributed in $[0,2\pi]$ and is independent of Y(t). Show that X(t) is WSS.
- 6 a) Consider the random process X(t) = A cos(ωt + θ) where A and ω are constants (7) and θ is a uniformly distributed random variable in (0,2π). Check whether or not the process is WSS.

(8)

b) The joint PDF of two continuous random variables X and Y is

$$f(x,y) = \begin{cases} 8xy, 0 < y < x < 1\\ 0, \quad otherwise \end{cases}$$

Α

i) Check whether X and Y are independent ii) Find P(X + Y < 1)

PART C

Answer any two full questions, each carries 20 marks

- 7 a) The number of particles emitted by a radioactive source is Poisson distributed. The source (4) emits particles at a rate of 6 per minute. Each emitted particle has a probability of 0.7 of being counted. Find the probability that 11 particles are counted in 4 minutes.
 - b) Assume that a computer system is in any one of the three states: busy, idle and under (8) repair, respectively, denoted by 0,1,2. Observing its state at 2 P. M. each day, the transition probability matrix is $P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \end{bmatrix}$

$$= \begin{bmatrix} 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix}$$

Find out the third step transition probability matrix and determine the limiting probabilities.

c) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 (8) per minute, find the probability that the interval between two consecutive arrivals is:

i) More than 1 minute ii) Between 1 minute and 2 minutes

iii) Less than or equal to 4minutes.

8 a) Use Trapezoidal rule to evaluate $\int_0^1 x^3 dx$ considering five subintervals (4)

Using Newton's forward interpolation formula, find y at x = 8 from the following b) (8) 5 20 25 table: **x** : 0 10 15 24 14 18 32 **v**: 7 11

c) Using Euler's method, solve for y at x = 0.1 from $\frac{dy}{dx} = x + y + xy$, y(0) = 1 (8) taking step size h = 0.025.

9 a) The transition probability matrix of a Markov chain $\{X_n, n \ge 0\}$ having three states (10) 1.2 and 2 is $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.2 \end{bmatrix}$ and the initial probability distribution is

1, 2 and 3 is
$$P = \begin{bmatrix} 0.1 & 0.6 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}$$
 and the initial probability distribution is

$$p(0) = [0.5 \quad 0.3 \quad 0.2]$$
. Find the following:

i) $P\{X_2 = 2\}$ ii) $P\{X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 3\}$.

- b) Using Newton-Raphson method, compute the real root of $f(x) = x^3 2x 5$ (5) correct to 5 decimal places.
- c) Using Lagrange's interpolation formula, find the values of y when x = 10 from (5) the following table :