A A2801 Pages: 2

Reg No.:______________ Name:______________



# APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SECOND SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

**Course Code: MA102**

**Course Name: DIFFERENTIAL EQUATIONS**

Max. Marks: 100 Duration: 3 Hours

## PART A

***Answer all questions, each carries 3 marks.***

| | | Marks |
|---|---|---|
| 1 | Solve the initial value problem $xy' = y - 1, y(0) = 1$ | (3) |
| 2 | Solve the following differential equation by reducing it to first order $xy'' = 2y'$. | (3) |
| 3 | Find the particular integral of $(D^2 + 3D + 2)y = 3$. | (3) |
| 4 | Find the particular integral of $y'' + y = \sin x$. | (3) |
| 5 | Obtain the Fourier series expansion for the function $f(x) = x$ in the range $-\pi < x < \pi$. | (3) |
| 6 | Find the Fourier sine series of the function $f(x) = \pi x - x^2$ in the interval $(0, \pi)$ | (3) |
| 7 | Form a partial differential equation by eliminating the arbitrary function in $xyz = \emptyset(x + y + z)$ | (3) |
| 8 | Solve $r + s - 2t = e^{x+y}$. | (3) |
| 9 | Solve one dimensional wave equation for $k < 0$. | (3) |
| 10 | Solve $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} - u = 0, u(x, 0) = 6e^{-3x}$ using method of separation of variables. | (3) |
| 11 | Find the steady state temperature distribution in a rod of length 30cm if the ends are kept at $20^0$C and $80^0$C. | (3) |
| 12 | Write down the possible solutions of one dimensional heat equation. | (3) |

## PART B

***Answer six questions, one full question from each module.***

### Module I

| | | | |
|---|---|---|---|
| 13 | a) | Verify that the given functions $x^{\frac{3}{2}}, x^{-\frac{1}{2}}$ are linearly independent and form a basis of solution space of given ODE $4x^2y'' - 3y = 0$. | (6) |
| | b) | Solve the boundary value problem: $$y'' - 10y' + 25y = 0, \quad y(0) = 1, \quad y(1) = 0.$$ | (5) |

**OR**

| | | | |
|---|---|---|---|
| 14 | a) | Find the general solution of $y'''' + 2y'' + y = 0$. | (6) |
| | b) | Find a fundamental set of solutions of $2t^2y'' + 3ty' - y = 0, t < 0$. Given that $y_1(t) = \frac{1}{t}$ is a solution. | (5) |

### Module II

| | | | |
|---|---|---|---|
| 15 | a) | Find the particular integral of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4cos^2x$. | (6) |
| | b) | Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$ using method of variation of parameters. | (5) |

**OR**

| | | | |
|---|---|---|---|
| 16 | a) | Solve $x^2y'' - xy' - 3y = x^2 \ln x$ | (6) |
| | b) | Solve $y'''' - 2y''' + 5y'' - 8y' + 4y = e^x$. | (5) |

## Module III

17 a) Obtain the Fourier series expansion of $f(x) = x \sin x$ in the interval $(-\pi, \pi)$. (6)

b) Find the half range sine series of $f(x) = k$ in the interval $(0, \pi)$. (5)

**OR**

18 a) Find the Fourier series of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $(0, 2\pi)$. (6)

b) Find the half range sine series of $f(x) = e^x$ in $(0,1)$. (5)

## Module IV

19 a) Solve $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 2\sin(3x + 2y)$ (6)

b) Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$ (5)

**OR**

20 a) Form the PDE by eliminating $a, b, c$ from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (6)

b) Solve $(x + y)zp + (x - y)zq = x^2 + y^2$. (5)

## Module V

21 A tightly stretched violin string of length 'a' and fixed at both ends is plucked at its mid-point and assumes initially the shape of a triangle of height 'h'. Find the displacement u(x,t) at any distance 'x' and any time 't' after the string is released from rest. (10)

**OR**

22 Solve the PDE $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. (10)

Boundary conditions are $u(0, t) = u(l, t) = 0,\ t \geq 0$

Initial conditions are $y(x, 0) = a\sin\left(\frac{\pi x}{l}\right)$ and $\frac{\partial y}{\partial t} = 0 \quad at \quad t = 0$.

## Module VI

23 A rod, 30 cm long has its ends A and B kept at $20^0$C and $80^0$C respectively, until the steady state conditions prevail. The temperature at each end is then suddenly reduced to $0^0$C and kept so. Find the resulting temperature function u(x,t) taking x=0 at A. (10)

**OR**

24 A long iron rod with insulated lateral surface has its left end maintained at a temperature $0^0$C and its right end at x=2, maintained at $100^0$C. Determine the temperature as a function of 'x' and 't' if the initial temperature is (10)

$$u(x, 0) = \begin{cases} 100x, & 0 < x < 1 \\ 100, & 1 < x < 2 \end{cases}$$

****