APJ ABDULKALAM TECHNOLOGICAL UNIVERSITY
08 PALAKKAD CLUSTER

Q. P. Code : 10171 **(Pages: 4)** **Name** .....

Reg. No: .....

FIRST SEMESTER M.TECH. DEGREE EXAMINATION December 2017

Branch: CS

08CS6041 MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

Time:3 hours **Max.marks: 60**

Answer all six questions.

Modules 1 to 6: Part 'a' of each question is compulsory and answer either part 'b' or part 'c' of each question.

Q.no.	Module 1	Marks
1.a	Determine the eigen values of $A=\begin{bmatrix} 1 & 0 & 4 \\ 0 & 4 & 0 \\ 3 & 5 & -3 \end{bmatrix}$	3
	Answer b or c	
b	Solve the following system of equations by LU decomposition method $3x + 2y + 7z = 4$ $2x + 3y + z = 5$ $3x + 4y + z = 7$	6
c	Find the eigen values and eigen vectors of A^TA wher $A=\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ .Also check $A=UDV^T$	6

Q.no.	Module 2	Marks
2.a	Define Baye's theorem	3

Answer b or c

b The contents of Urn 1,Urn 2, Urn3 are as follows,1 white,2 black and 3 red. 2 white,1 black and 1 red balls and 4 white, 5 black and 3 red balls respectively. One Urn is chosen at random and 2 balls are drawn from it. They happen to be white and red. What is the probability that they come from Urn1,2 or 3. **6**

c Assume that the prior distribution for the proportion of defectives by a machine is P. The no;of defectives among the random sample is 2.Find the posterior distribution of P. Given that x is the observer which follows binomial distribution. **6**

P	Π(P)
0.1	0.6
0.2	0.4

Q.no.	Module 3	Marks
3.a	Define Markov chain. What are the classifications of states of a Markov chain?	3

Answer b or c

b The one step TPM of a Markov chain { Xn : n=0,1,2,....} having state space S=(1,2,3) is **6**

$$\begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

And the initial distribution is π_0 =(0.7,0.2,0.1). Find

1) $P(X_2=3/X_0=1)$

2) $P(X_3=2, X_2=3, X_1=3, X_0=2)$

3)$P(X_2=3)$

c Let { Xn : n=0,1,2,....} be a Markov chain with state space S={0,1,2} and one step TPM **6**

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1 & 0 \end{bmatrix}$$

1) Is the chain Ergodic?

2) Find the Invariant probabilities. \

Q.no.	Module 4	Marks
4.a	Define Poisson Process	3
	Answer b or c	
b	Prove that the inter arrival time of a Poisson Process with parameter λ follows an exponential distribution with mean $1/\lambda$.	6
c	Suppose that the customers arrive at a bank according to a Poisson Process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes 1) Exactly 4 customers arrive 2) Less than 4 customers arrive 3) More than 4 customers arrive	6

Q.no.	Module 5	Marks
5.a	In the usual notation of a (M/M/1) : (∞/FIFO) queue system find P(N>2) if λ=12/hrs. and μ=30/hrs.	4
	Answer b or c	
b	A supermarket has 2 servers servicing customers at counters. The customers arrive in a Poisson fashion at the rate of 30/ hrs. The service time for each customer is expected with mean 4 minutes. Find the probability that a customer has to wait for the service, avg. queue length and the avg. time spent by a customer in the queue.	8
c	A beauty parlour shop with one beautician, ladies arrive according to a Poisson distribution with mean arrival rate of 5 per hour and a hair design was exponentially distributed with an avg. design taking in minutes.As it is a very good parlour, customers do have patients to wait. Find 1) Avg. No: of ladies in the shop and the avg. no: of waiting to do the hair design. 2) % of time an arrival can walk inside the parlour without having to wait. 3) % of ladies who has to wait prior to getting into the chair for hair design.	8

Q.no.	Module 6	Marks
6.a	Define Pollaczek- Khinchine formula	4
	Answer b or c	
b	Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson Process with mean of 4 cars per hour and may wait in facility's parking lot if the bay is busy. If the service time for all cars is constant and equal to 10 minutes. Determine Ls,Lq,Ws,Wq	8
c	A TVS company in Chennai containing a repair section shared by a large no: of machines has 2 sequential stations with respective service rates of 3 per hour & 4 per hour. The cumulative failure rate of all the machine is 1 per hour. Assuming that he system behaviour can be approximated by the 2 stage Tandem queue, find 1) Probability that both the service stations are idle(free) 2) The avg. repair time including the waiting time. 3) The bottleneck of the repair facility.	8