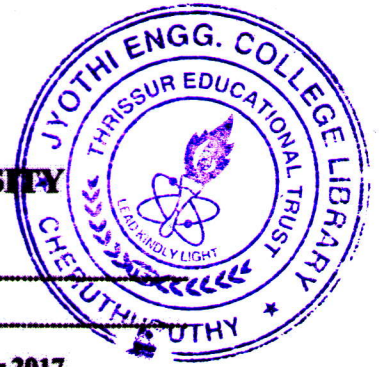


APJ ABDULKALAM TECHNOLOGICAL UNIVERSITY
08 PALAKKAD CLUSTER



Q. P. Code : 10171

(Pages: 4)

Name

Reg. No:

FIRST SEMESTER M.TECH. DEGREE EXAMINATION December 2017

Branch: CS

08CS6041 MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

Time: 3 hours

Max. marks: 60

Answer all six questions.

Modules 1 to 6: Part 'a' of each question is compulsory and answer either part 'b' or part 'c' of each question.

Q.no.	Module 1	Marks
1.a	Determine the eigen values of $A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 4 & 0 \\ 3 & 5 & -3 \end{bmatrix}$	3
	Answer b or c	
b	Solve the following system of equations by LU decomposition method	6
	$3x + 2y + 7z = 4$	
	$2x + 3y + z = 5$	
	$3x + 4y + z = 7$	
c	Find the eigen values and eigen vectors of $A^T A$ where $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Also check $A = UDV^T$	6
Q.no.	Module 2	Marks
2.a	Define Baye's theorem	3

Answer b or c

- b The contents of Urn 1, Urn 2, Urn 3 are as follows, 1 white, 2 black and 3 red. 2 white, 1 black and 1 red balls and 4 white, 5 black and 3 red balls respectively. One Urn is chosen at random and 2 balls are drawn from it. They happen to be white and red. What is the probability that they come from Urn 1, 2 or 3. 6
- c Assume that the prior distribution for the proportion of defectives by a machine is P. The no. of defectives among the random sample is 2. Find the posterior distribution of P. Given that x is the observer which follows binomial distribution. 6

P	$\Pi(P)$
0.1	0.6
0.2	0.4

Q.no.	Module 3	Marks
3.a	Define Markov chain. What are the classifications of states of a Markov chain?	3

Answer b or c

- b The one step TPM of a Markov chain $\{X_n : n=0,1,2,\dots\}$ having state space $S=(1,2,3)$ is 6

$$\begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

And the initial distribution is $\pi_0=(0.7,0.2,0.1)$. Find

- 1) $P(X_2=3/X_0=1)$
 - 2) $P(X_3=2, X_2=3, X_1=3, X_0=2)$
 - 3) $P(X_2=3)$
- c Let $\{X_n : n=0,1,2,\dots\}$ be a Markov chain with state space $S=\{0,1,2\}$ and one step TPM 6

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1 & 0 \end{bmatrix}$$

- 1) Is the chain Ergodic?
- 2) Find the Invariant probabilities. \

Q.no.	Module 4	Marks
4.a	Define Poisson Process	3
Answer b or c		
b	Prove that the inter arrival time of a Poisson Process with parameter λ follows an exponential distribution with mean $1/\lambda$.	6
c	Suppose that the customers arrive at a bank according to a Poisson Process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes	6
	1) Exactly 4 customers arrive	
	2) Less than 4 customers arrive	
	3) More than 4 customers arrive	

Q.no.	Module 5	Marks
5.a	In the usual notation of a (M/M/1) : (∞ /FIFO) queue system find $P(N > 2)$ if $\lambda = 12/\text{hrs}$. and $\mu = 30/\text{hrs}$.	4
Answer b or c		
b	A supermarket has 2 servers servicing customers at counters. The customers arrive in a Poisson fashion at the rate of 30/ hrs. The service time for each customer is expected with mean 4 minutes. Find the probability that a customer has to wait for the service, avg. queue length and the avg. time spent by a customer in the queue.	8
c	A beauty parlour shop with one beautician, ladies arrive according to a Poisson distribution with mean arrival rate of 5 per hour and a hair design was exponentially distributed with an avg. design taking in minutes. As it is a very good parlour, customers do have patients to wait. Find	8
	1) Avg. No: of ladies in the shop and the avg. no: of waiting to do the hair design.	
	2) % of time an arrival can walk inside the parlour without having to wait.	
	3) % of ladies who has to wait prior to getting into the chair for hair design.	

Q.no.

Module 6

Marks

6.a Define Pollaczek- Khinchine formula

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Answer b or c

b Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson Process with mean of 4 cars per hour and may wait in facility's parking lot if the bay is busy. If the service time for all cars is constant and equal to 10 minutes. Determine L_s, L_q, W_s, W_q

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c A TVS company in Chennai containing a repair section shared by a large no: of machines has 2 sequential stations with respective service rates of 3 per hour & 4 per hour. The cumulative failure rate of all the machine is 1 per hour. Assuming that the system behaviour can be approximated by the 2 stage Tandem queue, find

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- 1) Probability that both the service stations are idle(free)
- 2) The avg. repair time including the waiting time.
- 3) The bottleneck of the repair facility.