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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017
Course Code: MA201
Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100   Duration: 3 Hours

PART A
Answer any two full questions, each carries 15 marks.   Marks

1 a) Find the points where Cauchy-Riemann equations are satisfied for the function   (7)
      $f(z) = xy^2 + i\, x^2\, y$. Where does $f^{|}(z)$ exist? Is the function $f(z)$ analytic at those
      points?

   b) If $v = e^x (x \sin y + y \cos y)$, find an analytic function $f(z)=u+iv$.   (8)

2 a) Show that $u = x^2-y^2-y$ is harmonic. Also find the corresponding conjugate harmonic   (7)
      function.

   b) (i) Find a bilinear transformation which maps $(-i, 0, i)$ onto $(0, -1, \infty)$.   (8)

      (ii) Test the continuity at $z = 0$, if $f(z) = \dfrac{Im\, z}{|z|}, z \neq 0$

                              $= 0, z = 0$

3 a) Find the image of the lines x=1, y=2 and x>0, y<0 under the mapping $W= z^2$ (8)

   b) Find the image of the semi-infinite strip $x > 0, 0< y < 2$ under the transformation   (7)
      w=iz+1. Draw the regions.

PART B
Answer any two full questions, each carries 15 marks.

4 a) Evaluate $\oint Re\, z^2 dz$ over the boundary C of the square with vertices 0, i, 1+ i,1   (8)
      clockwise

   b) Evaluate $\int \dfrac{4-3z}{z(z-1)}$ dz over the circle $|z|=\dfrac{3}{2}$ (4)

   c) Evaluate $\int \dfrac{3z^2+7z+1}{z+1}$ dz over the circle $|z+ i |=1$ (3)

5 a) Expand $\dfrac{z}{(z-1)(z-2)}$ in (1) $0<|z-2|<1$, (2) $|z-1|>1$ (8)

   b) Evaluate $\int_0^{2\pi} \dfrac{1}{2+\cos\theta}\, d\theta$ (7)

6 a) Using Residue theorem evaluate $\int \dfrac{z^2}{(z-1)^2(z+2)}$ dz over the circle $|z|=3$ (7)

   b) Find the Taylor series of $\dfrac{\sin z}{z-\pi}$ about the point $z= \pi$ (4)

c)   Evaluate $\int \frac{\sin z}{z^6} dz$ over the circle $|z|=2$ using Cauchy's Residue theorem.   (4)

PART C
Answer any two full questions, each carries 20 marks.

7. a)   Solve by Gauss-Elimination method $x + y + z = 6$, $x+ 2y- 3z = -4$, $-x-4y+9z =18$.   (7)

b)   Find the values of 'a' and 'b' for which the system of equations $x + y + 2z =2$,

$2x-y+3z=10, 5x-y+az=b$ has: (7)

    (i) no solution   (ii) unique solution   (iii) infinite number of solutions.

c)   Verify whether the vectors $(1,2,1,2)$, $(3,1,-2,1)$,$(4,-3,-1,3)$ and $(2,4,2,4)$ are linearly

independent in R^4 .   (6)

8   a)   Write down the matrix associated with the quadratic form $8x_1^2+7x_2^2+3x_3^2-12x_1x_2$

$-8x_2x_3+4x_3x_1$. By finding eigen values, determine nature of the quadratic form.   (7)

b)   Diagonalise the matrix $A = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 0 & 2 \\ 0 & 2 & -1 \end{bmatrix}$ (7)

c)   If A is a symmetric matrix, verify whether AA^T and A^TA are symmetric?   (6)

9   a)   Find the eigen vectors of $A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$ (8)

b)   Find the null space of $AX=0$ if $A=\begin{bmatrix} 1 & 1 & 0 & 2 \\ -2 & -2 & 1 & -5 \\ 1 & 1 & -1 & 3 \\ 4 & 4 & -1 & 9 \end{bmatrix}$ (6)

c)   Verify whether $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.   (6)

What can you say about determinant of an orthogonal matrix? Prove or disprove the

result.

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