SIXTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, APRIL 2017

Computer Science Engineering CS 14 605-GRAPH THEORY AND COMBINATORICS



Time : Three Hours
Maximum : 100 Marks

## Part A

Answer any eight questions.
Each question carries 5 marks.

1. Define a planar graph. Show that $\mathrm{K}_{5}$ is not a planar graph.
2. Find, if possible, an Euler trail or a semi-Euler trail in this graph :

3. How many different Hamilton's paths are there for $\mathrm{Kn}, \mathrm{n}$ and $n=1$ ?
4. Suppose that a graph G is regular of degree $r$, where $r$ is odd :
(i) Prove that $G$ has an even number of vertices.
(ii) Prove that the number of edges in G is a multiple of $r$.
5. Discuss about weighted tress and prefix codes.
6. Determine the number of positive integers $n$ where $1 \leq n \leq 200$ and $n$ is not divisible by $2,3,5$.
7. Define (i) Cutset ; (ii) Edge connectivity ; (iii) Vertex connectivity with one example each.
8. In how many ways can one arrange the letters in APPLE so that :
(i) There is no pair of consecutive identical letters.
(ii) There are exactly two pairs of consecutive identical letters.
9. Find the generating function of $a_{n}+a_{n-1}-6 a_{n-2}$ for $n \geq 2, a_{0}=-1$ and $a_{1}=8$.
10. Give the generating function for $1,1,1,1, \ldots 1,0,0,0 \ldots$ first terms are 1 , others are 0 .
( $8 \times 5=40$ marks )

## Part B <br> Answer all questions. <br> Each question carries 15 marks.

1. (a) Show that in any connected planar graph with $n$ vertices, $e$ edges and $f$ faces $e-n+2=f$ (Euler's formula).

## Or

(b) Explain with an example, Chinese postman problem.
2. (a) Prove that the maximum flow possible between two vertices $a$ and $b$ in a network is equal to the minimum of the capacities of all cut-sets with respect to $a$ and $b$.
Or
(b) Explain in detail biconnected components and articulation points of tree.
3. (a) In how many ways can the integers $1,2,3, \ldots 10$ be arranged in a line so that no even integer is in its natural place.

## Or

(b) Illustrate binomial theorem with neat example.
4. (a) Solve linear recurrence relation :

$$
{ }^{\prime} C_{n}=3 C_{n-1}-2 C_{n-2} \text { with } C_{1}=5, C_{2}=3
$$

Or
(b) Determine the sequence generated by exponential generating function :

$$
\begin{aligned}
& e^{2 x}-3 x^{3}+5 x^{2}+7 x \\
& (3+x)^{3}
\end{aligned}
$$

$$
(4 \times 15=60 \text { marks })
$$

