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Reg. No.:____________________ Name:____________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE SPECIAL EXAMINATION, AUGUST 2016

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer ALL questions. Each question carries 3 marks

1. Find derivative of y = sinh (4x-8)
2. Test whether the series converges or diverges, $\sum_{k=1}^{\infty}\frac{k}{2^k}$
3. Identify the surface $z = y^2 - x^2$
4. Convert from rectangular to spherical co-ordinates, $(2\sqrt{3}, 2, -4)$
5. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $Z = \cos\ (xy^3)$
6. Show that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ if $z = x^2y + 5y^3$.
7. Evaluate $\int_0^2 (2t\hat{\imath} + 3t^2\hat{\jmath})dt$
8. Find the arc length of the parametric curve $x=e^t$, $y=e^{-t}$,$z = \sqrt{2}\,t$,$0 \le t \le 1$.
9. Evaluate $\int_1^3 \int_2^4 (40 - 20\ xy\ )\ dy\ dx$
10. Evaluate $\int_0^3 \int_0^2 \int_0^1 (xyz\ )\ dx\ dy\ dz$

PART B

Answer any 2 complete questions each having 7 marks

11. Test the convergence of the series $\sum_{k=1}^{\infty} \frac{k(k+3)}{(k+1)(k+2)(k+5)}$
12. Show that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$
13. Find the Taylor series of $\frac{1}{x+2}$ about x =1.

Answer any 2 complete questions each having 7 marks

14. Express the equation $x^2 - y^2 - z^2 = 0$ in cylindrical and spherical coordinates.

15. Evaluate $\lim_{(x,y)\to(0,0)} [\sin\ (\sqrt{x^2+y^2})]/(x^2+y^2)$ by converting to polar coordinates.
16. Show that the functions $f(x, y) = 3x^2y^5$ and $f(x, y) = \sin(3x^2y^5)$ are continuous everywhere.

Answer any 2 complete questions each having 7 marks

17. Let L(x, y) denote the local linear approximation to $f(x, y) = \sqrt{x^2+y^2}$ at the point (3, 4). Compare the error in approximating $f(3.04, 3.98) = \sqrt{(3.04\ )^2 + (3.98\ )^2}$ by L (3.04, 3.98) with the distance between the points (3,4) and (3.04, 3.98).
18. Suppose that $w = x^2 + y^2 - z^2$ and $x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$, $z = \rho \cos\phi$. Use appropriate form of the chain rule to find $\frac{\partial w}{\partial \rho}$ and $\frac{\partial w}{\partial \theta}$
19. Locate the relative extrema and saddle points of $f(x,y) = 3x^2 - 2xy + y^2 - 8y$

Answer any 2 complete questions each having 7 marks

20. Let $f(x,y) = x^2e^y$. Find the maximum value of a directional derivative at (-2,0) and find the unit vector in the direction in which the maximum value occur.
21. Find the angle between the tangent lines to the graphs of $r_1(t) = \tan^{-1} t\,i + \sin t\, j + t^2 k$ $r_2(t) = (t^2 - t)i + (2t-2)j + \log t\, k$
22. Suppose that a particle moves through 3-space so that its position vector at time

    t is $r(t) = ti + t^2 j + t^3 k$.

    Find the scalar tangential and normal components of acceleration at time $t = 1$.

Answer any 2 complete questions each having 7 marks

23. Use a polar double integral to find the area enclosed by the circle $r = \sin\theta$
24. Use a triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $z = 5$
25. Evaluate $\iint_R \frac{x-y}{x+y}\, dA$ where R is the region enclosed by $x - y = 0, x - y = 1, x + y = 1,$

    $x + y = 3$