C 1241 (Pages : 3) Name.....

Reg. No.....

FOURTH SEMESTER B.TECH. (ENGINEERING) [14 SCHEME] DEGREE EXAMINATION, APRIL 2016

EN 14 401 (B)—ENGINEERING MATHEMATICS—IV

(Common for IC, EC, EE, AI, BM, CS and IT)

Time : Three Hours Maximum : 100 Marks

Part A

I. Answer any *eight* questions out of ten :

1 Let the p.d.f. of a continuous random variable X be $f(x)=\alpha e^{-|x|}, -\infty<x<\infty$. Find α. Calculate the mean and the variance of X.

2 How many families would be expected to have :

(i) 2 boys and 2 girls. (ii) at least one boy.

(iii) atmost 2 girls. (iv) children of both sexes.

out of 1000 families with 4 children each. Assume equal probabilities for boys and girls.

3 Find the Z-transform of $x(n)=\left(\frac{1}{2}\right)^n u(-n)+\left(\frac{1}{3}\right)^n u(-n)$.

4 Find the inverse Z-transform of $X(z)=\frac{1}{1-az^{-1}}, |z|>a$.

5 Prove that $J_{-3/2}(x)=-\sqrt{\frac{2}{\pi x}}\left(\sin x-\frac{\cos x}{x}\right)$.

6 Prove that $J_n'(x)+\frac{n}{x}J_n(x)=J_{n-1}(x)$.

7 Solve $z^2\left(p^2+q^2+1\right)=a^2$.

8 Solve the differential equation :

$(mz-ny)p+(nx-lz)q=ly-mx$.

9 Classify the PDE $x^2u_{xx}+\left(1-y^2\right)u_{yy}=0$ for $-1<y<1, -\infty<x<\infty$.

10 If 0.8% of the fuses delivered to a factory are defective, find the probability that atmost 4 fuses will be defective in a random sample of 400 fuses.

(8 × 5 = 40 marks)

Turn over

Part B

II. Answer *all* questions :

11. (a) A taxi cab company has 12 Ambassadors and 8 Fiats. If 5 of these taxi cabs are in the workshop for repairs and an Ambassador is as likely to be in for repairs as a Fiat, what is the probability that :

(i) 3 of them are Ambassadors and 2 are Fiats.

(ii) at least 3 of them are Ambassadors and

(iii) all the 5 are of the same make.

(b) Buses arrive at a specified stop at 15 min intervals starting at 7 a.m. that is they arrive at 7, 7:15, 7:30, 7:45 and so on. If a passenger arrives at the stop at a random time that is uniformly distributed between 7 and 7:30 a.m. find the probability that he waits (i) less than 5 minutes for a bus ; and (ii) at least 12 min for a bus.

Or

12. (a) In an engineering examination, a student is considered to have failed, secured second class, first class and distinction according as he scores less than 45%, between 45% and 60% between 60% and 75% and above 75% respectively. In a particular year 10% of the student failed in the examination and 5% of the students got distinction. Find the percentages of students who have got first class and second class (Assume normal distribution of marks).

(b) In a certain city, the daily consumption of electric power in millions of Kilowatt-hours can be treated as a RV having a Gamma distribution with parameter $\lambda = 1/2$ and $k = 3$. If the power plant of this city has a daily capacity of 12 millions Kilowatt-hours, what is the probability that this power supply will be inadequate on any given day.

13. (a) Find the Z-transform of the signal $x(n) = [\sin w_0 n]u(n)$ and find ROC.

(b) If $x(n) = x_1(n) * x_2(n)$ where $x_1(n) = \left(\frac{1}{2}\right)^n u(n)$ and $x_2(n) = \left(\frac{1}{3}\right)^n u(n)$, find X (z) by using convolution property for Z-transform.

Or

14. (a) Find the inverse Z-transform of $X(z) = \dfrac{1+2z^{-1}}{1-2z^{-1}+z^{-2}}$ if $x(n)$ is (i) causal ; (ii) anti-causal using long division.

(b) Find Z-transform of the sequence :

$x(n) = \{1, 2, 0, -4, 3, 2, 1, 6, 5\}$.

15. (a) Prove that $J_{-n}(x) = (-1)^n J_n(x)$.

(b) Solve in series the equation $\frac{d^2y}{dx^2} + y = 0$.

Or

16. Find the solution in series of the differential equation :

$2x\frac{d^2y}{dx^2} + (1-2x)\frac{dy}{dx} - y = 0$ by Frobenius method.

17. (a) Derive the one-dimensional wave equation.

(b) Solve the following PDE's.

(i) $\sqrt{p} + \sqrt{q} = x + y$.

(ii) $p = e^q$.

(iii) $(p+q)(z - px - qy) = p^2 - q^2$.

Or

18. (a) Solve the one-dimensional heat equation by the method of separation of variables.

(b) Solve the differential equation :

$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$.

(4 × 15 = 60 marks)