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FOURTH SEMESTER B.TECH. (ENGINEERING) [14 SCHEME] DEGREE EXAMINATION, APRIL 2016

EN 14 401 (A)—ENGINEERING MATHEMATICS—IV

(Common for ME, CE, PE, CH, BT, MT, PT, AM and AN)

Time : Three Hours Maximum : 100 Marks

Part A

I. Answer any *eight* questions out of ten :

1 If the p.d.f. of a continuous random variable X is given by :

$$f(x) = \begin{cases} k\left(1-x^2\right) & \text{for } 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find (a) the value of K ; and (b) the probability $P(0.5 \le X \le 2)$; and (c) mean of the random variable.

2 If the probability of hitting a target is 10% and 10 shots are fired independently, what are the probabilities that the target will be hit (i) twice ; (ii) at least once ; and (iii) atmost once.

3 A shipment of 15 tape-recorders contains 4 that are defective. If 8 of them are randomly selected for inspection, what is the probability that atmost two of them are defective ?

4 According to norms established for a reading comprehension test, eighth graders should average 83.3 with a standard deviation of 8.6. If 45 randomly selected eighth graders from a certain school district averaged 87.8, test the null hypothesis $\mu = 84.3$ against $\mu > 84.3$ at 0.01 level of significance.

5 35 determinations of the thermal conductivity of a certain kind of brick yielded an average value of 0.343. Test the hypothesis that the thermal conductivity of such a brick is 0.340 at 0.05 level of significance, assuming that the variability of such determinations is 0.01.

6 Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

7 Prove $J_{-n}(x) = (-1)^n J_n(x)$.

8 Form the partial differential equation by eliminating the arbitrary constants a, b, c from $z = ax^2 + bxy + cy^2$.

Turn over

9 Solve the partial differential equation $\log\left(\frac{\partial^2 z}{\partial x^2}\right) = x + y$.

10. Solve $9\left(p^2 z + q^2\right) = 4$.

(8 × 5 = 40 marks)

Part B

II. Answer *all* questions :

11. (a) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.

(b) Six coins are tossed 6,400 times. Using Poisson distribution, determine the approximate probability of getting six heads x times.

(c) If the probability that a burglar will be caught in any given job is 0.20. Find the probability that he will be caught for the first time on his fourth job.

Or

12. (a) In a certain factory producing razor blades, there is a small chance $\frac{1}{500}$ for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no. defective blade ; (ii) at least 1 defective blade ; and (iii) atmost 1 defective blade in a consignment of 10,000 packets.

(b) In a certain city, the daily consumption of electric power in millions of Kilowatt-hours can be treated as a RV having Gamma distribution with parameter $\lambda = \frac{1}{2}$ and $k = 3$. If the power plant of this city has a daily capacity of 12 millions Kilowatt-hours. What is the probability that this power supply will be inadequate on any given day.

(c) Fit a Binomial distribution to the following data :

x	:	0	1	2	3	4	5
f	:	2	14	20	34	22	8

13. (a) The following samples are measurements of the heat producing capacity of specimens of coal from two mines :

Mine I	:	8260	8130	8350	8070	8340	
Mine II	:	7950	7890	7900	8140	7920	7840

Use 0.01 level of significance to test whether the difference between the means of two samples is significant.

(b) For the following data, test $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$ at 10% level of significance

$n_1 = n_2 = 8, s_1^2 = 3.89, s_2^2 = 4.02$.

Or

14. (a) Fit a Poisson distribution to the following data and test the goodness of fit at the 1% level of significance :

x	:	0	1	2	3	4
f	:	419	352	154	56	19

(b) A sample of 10 measurements of the diameter of a sphere gave a mean of 12 cm and a standard deviation of 0.15 cm. Find a (i) 95% and (ii) 99% confidence interval for the actual diameter.

15. (a) Solve in series the equation $\frac{d^2y}{dx^2} + y = 0$.

(b) Prove that (i) $J_{v-1}(x) + J_{v+1}(x) = \frac{2v}{x} J_v(x)$; (ii) $J_{v-1}(x) - J_{v+1}(x) = 2J_v'(x)$.

Or

16. Find the solution in series of the differential equation $2x\frac{d^2y}{dx^2} + (1-2x)\frac{dy}{dx} - y = 0$ by Frobenius method.

17. (a) Solve the differential equation :

$$(2z - y)p + (x + z)q + 2x + y = 0.$$

(b) Derive the one dimensional heat equation.

Or

18. (a) Solve the one dimensional wave equation by the method of separation of variables.

(b) Solve the PDE's :

(i) $2(p^2 - q^2) = 3pq$.

(ii) $p^2q^2 + x^2y^2 = x^2q^2(x^2 + y^2)$.

(iii) $(px + qy - z)^2 = 1 + p + q$.

(4 × 15 = 60 marks)