APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH. DEGREE EXAMINATION DECEMBER 2015

Computer Science and Engineering

08CS6041 MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

Time:3 hours **Max.marks: 60**

Answer **ALL** six questions. Part '**A**' of each question is compulsory.
Answer **EITHER** part '**B**' or part '**C**' of each question.

Q.no.	Module 1	Marks
1.A	Define vector space. Give an example	3
	Answer B or C	
B	Find an LU factorization of $A=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 3 & -3 & 5\end{bmatrix}$	
C	Determine the eigen values of $A=\begin{bmatrix}1 & 0 & 4\\ 0 & 4 & 0\\ 3 & 5 & -3\end{bmatrix}$	6

Q.no.	Module 2	Marks
2.A	State and prove Baye's theorem.	3
	Answer B or C	
B	A box has 100 tickets marked with numbers 1 to 100. One ticket is drawn at random from the box. Find the probability that the number on the drawn ticket is (i) less than 20 (ii) even (iii) a multiple of 5.	6
C	The probabilities of X,Y and Z becoming managers re 4/9,2/9 and 1/3 respectively. The probabilities that the bonus scheme will be introduced if X, Y and Z become managers are 3/10,1/2 and 4/5 respectively. If the bonus scheme is introduced, what is the probability that X is the manager?	6

Q.no.	Module 3	Marks
3.A	State and prove Chapman Kolmogrov equation?	3

Answer B or C

B	The transition probability matrix of Markov chain $\{X_n\}$,n=1,2,3,,,,,having 3 states 1,2,and 3 is $P=\begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ And theinitial distribution is $P^{(0)}=(0.7,0.2,0.1)$.Find (i)$P(X_2=3)$and (ii)$P(X_{3=}2,X_2=3,X_1=3,X_0=2)$	6
C	The process$\{X(t)\}$whose probability distribution under certain condition is given by $$P(X(t)=n)=\begin{cases}\frac{(at)^{n-1}}{(1+at)^{n+1}}, n=1,2,.. \\ \frac{at}{1+at}, n=0\end{cases}$$ Show that it is not stationary.	6

Q.no.	Module 4	Marks
4.A	Derive birth and death process?	3

Answer B or C

B	The arrival of cars in a toll booth is a poisson process with a mean arrival rate of 2 per hour. (i)Find the probability that exactly 4 cars arrive in 2 hour period. (ii)Find the probability that 6 or more cars arrive in 2 hour period.	6
C	Show that the Poisson process is Evolutionary?	6

Q.no.	Module 5	Marks
5.A	Briefly explain simple Markovian Queues.	4

Answer B or C

B 8

Arrivals at a telephone booth are considered to be poisson with an average time of 12 min.between one arrival and the next.The length of a phone call is assumed to be distributed exponentially with mean 4 min.

a)Find the average number of persons waiting in the system.
b)What is the probability that a person arriving at the booth will have to wait in the queue?
c) What is the probability that it will take him more than 10 min altogether to wait for the phone and complete his call?
d)Estimate the fraction of the day when thw phone will be in use.
e)The telephone dept. will install a second booth,when convinced that an arrival has to wait on the average for atleast 3 mins for phone.By how much the flow of arrivals should increase in order to justify a second booth?
f)What is the average length of the queue that forms from time to time?

C Suppose there are 3 typists pool. Each typist can type an average of 6 letters/hr. If letters arrive to be types at the rate of 15 letters/hr. 8

(i) What fraction of time are all 3 typists busy?
(ii) What is the average no.of letters waiting to be typed?
(iii) What is the average time a letter spends in the system?
(iv) What is the probability a letter will take longer than 20 minutes waiting to be typed and being typed?

Q.no.	Module 6	Marks
6.A	State Pollaczek-Khinchine formula?	4

Answer B or C

B	Explain M/G/1 Queueing model?	8
C	Write short notes on(i) Types of blocking (ii) Aggregating Markovian status.	8