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THIRD SEMESTER B.TECH. (ENGINEERING) [14 SCHEME] DEGREE EXAMINATION, NOVEMBER 2015

CS/IT 14 304—DISCRETE COMPUTATIONAL STRUCTURES

Time : Three Hours

Maximum : 100 Marks

Part A

Answer any **eight** *questions.*

1. Let p, q, r denote the following statements about a particular triangle ABC.

   p : Triangle ABC is isosceles.

   q : Triangle ABC is equilateral.

   r : Triangle ABC is equiangular.

   Translate each of the following into an English sentence :–

   (a) $q \to p$.

   (b) $q \leftrightarrow p$.

   (c) $\neg p \to \neg q$.

   (d) $p \wedge \neg q$.

   (e) $r \to p$.

2. Let n be an integer. Prove that n is even if and only if $31n + 12$ is even.

3. Write the following statement as an implication in two ways, each in the if-then form : Either Kamala practices her piano lessons or she will not go to the movies.

4. If A = {1, 2, 3}, and B = {2, 4, 5}, determine the following :

   (a) $|A \times B|$; (b) the number of relations from A to B ; (c) the number of relations on A ; (d) The number of relations from A to B that contain (1, 2) and (1, 5) ; (e) the number of relations from A to B that contain exactly five ordered pairs; and (f) the number of relations on A that contain at least seven elements.

5. Logic chips are taken from a container, tested individually, and labelled defective or good. The testing process is continued until either two defective chips are found or five chips are tested in total. Using a tree diagram, exhibit a sample space for this process.

Turn over

6. For each of the following functions, $f: Z \rightarrow Z$, determine whether the functions are one-to-one and whether it is onto. If the function is not onto, determine the range of $f(z)$.

(i) $f(x) = x + 7$.

(ii) $f(x) = 2x - 3$.

(iii) $f(x) = -x + 5$.

(iv) $f(x) = x^2$.

(v) $f(x) = x^2 + x$.

7. (a) If H, K are subgroups of a group G, prove that $H \cap K$ is also a subgroup of G.

(b) Give an example of a group G with sub-groups H, K such that $H \cup K$ is not a sub group of G.

8. Find all generators of the cyclic groups $(Z_{12}, +)$, $(Z_{16}, +)$ and $(Z_{24}, +)$.

9. If a_n, $n \geq 0$, is the unique solution of the recurrence relation $a_{n+1} - da_n = 0, a_3 = 153/49$, $a_5 = 1377/2401$, What is d ?

10. If Laura invests Rs. 2000 at 6 % interest compounded quarterly, how many months must she wait for her money to double ?

(8 × 5 = 40 marks)

Part B

11. (a) For primitive statements p, q, :

(i) Verify that $p \rightarrow [q \rightarrow (p \wedge q)]$ is a tautology.

(ii) Verify that $(p \vee q) \rightarrow [q \rightarrow q]$ is a tautology by using the result of part (i) along with the substitution rules and laws of logic.

(iii) Is $(p \vee q) \rightarrow [q \rightarrow (p \wedge q)]$ a tautology ?

Or

(b) The following are the valid arguments. Establish the validity of each by means of a truth table. In each case, determent which rows of the table are crucial for assessing the validity of the argument and which rows can be ignored.

(i) $[[p \wedge q) \rightarrow r] \wedge \neg q \wedge (p \rightarrow \neg r)] \rightarrow (\neg p \vee \neg q)$.

(ii) $[[p \vee (q \vee r)] \wedge \neg q] \rightarrow (p \wedge r)$.

12. (a) Let $a_1, a_2, a_3, \ldots\ldots$ be the integer sequence defined recursively by, (1) $a_1 = 1$; and (2) for all $n \in Z^+$ where $n \geq 2$, $a_n = 2a_{[n/2]}$.

(i) Determine a_n for all $2 \leq n \leq 8$.

(ii) Prove that $a_n \leq n$ for all $n \in Z^+$.

Or

(b) For $x = \{0.1\}$, let $a = X \times X$. Define the relation $\Re$ on A by (a, b) $\Re$ (c, d) if (i) $a < c$; or (ii) $a = c$ and $b \leq d$.

1. Prove that $\Re$ is a partial order of A.
2. Determine all minimal and maximal elements for this partial order.
3. Is there a least element ? Is there a greatest element ?
4. Is this partial order a total order ?

13 (a) Let G = S4. (i) For $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$, find the sub group $H = \langle \alpha \rangle$; (ii) Determine the left co-sets of H in G.

Or

(b) In S_5, find an element of order n, for all $2 \leq n \leq 5$. Also determine the (cyclic) sub-group of S_5 that each of these elements generates.

14. (a) (i) Find and solve a recurrence relation for the number of ways to park motorcycles and compact cars in a row of n spaces, if each cycle requires one space and each compact needs two. (All cycles are identical in appearance and as the cars are).

(ii) Also solve if (1) the motor cycles come in two distinct models and ; (2) the compact cars come in three different colours.

Or

(b) An alphabet Σ consists of the four numeric characters 1, 2, 3, 4 and seven alphabetic characters a, b, c, d, e, f, g. Find and solve a recurrence relation for the number of words of length n, where there are no consecutive (identical or distinct) alphabetic characters.

(4 × 15 = 60 marks)