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Name.............................

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THIRD SEMESTER B.TECH. [ENGINEERING] (14 SCHEME) DEGREE EXAMINATION, NOVEMBER 2015

EN 14 301—ENGINEERING MATHEMATICS—III

(Common for all Branches)

Time : Three Hours Maximum : 100 Marks

Part A

Answer any **eight** *questions.*
Each question carries 5 *marks.*

1. Prove that $w = \sin z$ is an entire function. If so find $\frac{dw}{dz}$.

2. Show that $e^x (x \cos y - y \sin y)$ is a harmonic function.

3. Find and graph the image of $-1 \le x \le 1, -\pi < y < \pi$ under the mapping $w = e^z$.

4. Prove that $\oint_C (z - z_0)^m \, dz = \begin{cases} 2\pi i & \text{if} \quad m = -1 \\ 0 & \text{if} \quad m \neq -1 \end{cases}$ and integer.

5. Using Cauchy's integral formula, evaluate $\int_C \frac{z}{(z-1)(z-2)^2} dz$ where C is $|z-2| = \frac{1}{2}$.

6. Find the poles and residues of $\frac{9z + i}{z + z^3}$.

7. Express $v = (1, -2, 5)$ in R^3 as a linear combination of the vectors $u_1 = (1, 1, 1), u_2 = (1, 2, 3)$ and $u_3 = (2, -1, 1)$.

8. Let W be the subspace of R^5 generated by the vectors $u = (1, 2, 3, -1, 2)$ and $v = (2, 4, 7, 2, -1)$. Find a basis of the orthogonal complement $W^{\perp}$ of W.

9. Find the Fourier sine integral representation of $f(t) = e^{-at}, 0 < t < \infty, a > 0$.

10. Find the Fourier cosine transform of the function $f(x) = \begin{cases} \cos x & , \quad 0 < x < a \\ 0 & , \quad x > a \end{cases}$.

(8 × 5 = 40 marks)

Turn over

Part B

Answer **all** *questions.*
Each question carries 15 *marks.*

11. (a) Find the analytic function whose real part is $\dfrac{\sin 2x}{\cosh 2y + \cos 2x}$.

(b) Prove that an analytic function with constant modulus is a constant.

Or

12. (a) Discuss the transformation $w = z + \dfrac{1}{z}$. What are its fixed points. What are the critical points ? Show that the transformation maps the circle $|z| = c$ into an ellipse. Discuss the case when $c = 1$.

(b) Find the Möbius transformation that maps the points $(2, i, -2)$ into the points $(1, i, -1)$.

13. (a) Using Cauchy's residuè theorem evaluate $\oint_C \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$ where C is the circle $|z| = 2$.

(b) Expand $\dfrac{1}{z(z-1)(z-2)}$ in the region (i) $|z| > 2$; (ii) $|z| < 1$; (iii) $1 < |z| < 2$.

Or

14. (a) Evaluate $\int_0^{2\pi} \dfrac{\sin \theta}{3 + \cos \theta} d\theta$; (b) Evaluate $\int_{-\infty}^{\infty} \dfrac{x^2}{(x^2+4)(x^2+9)} dx$.

15. (a) Find an orthonormal basis for the subspace spanned by $(1,1,1,1), (1,2,4,5)$ and $(1,-3,-4,-2)$ in R^4.

(b) Find *a*, *b*, *c* such that $(2,1,-1), (a,1,-1)$ and $(b,3,c)$ form an orthogonal basis of R^3.

Or

16. (a) Define an inner product space. Let $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Determine whether $< x, y > = x_1 y_1 - x_1 y_2 - x_2 y_1 + x_2 y_2$. Defines an inner product in R^2.

(b) State Schwartz's Inequality and triangle Inequality. Using the standard inner product verify them for the vectors $x = (-2, 3, 1)$ and $y = (3, -4, -1)$ in R^3.

17. (a) Find a Fourier cosine and sine integral representation of the function

$$f(t)=\begin{cases}\cos t & , \quad 0 \le t \le \pi/2 \\ 0 & , \quad t > \pi/2.\end{cases}$$

(b) If $\mathcal{F}\{f(t)\} = F(w)$ then show that $\mathcal{F}\{f(t-t_0)\} = e^{-iwt_0}\, F(w)$.

Or

18. (a) Find the Fourier integral representation of $f(t)=\begin{cases}1-t^2 & , \quad |t|<1 \\ 0 & , \quad |t|>1\end{cases}$. Hence evaluate

$$\int_0^{\infty} \frac{\sin x - x\cos x}{x^3} \cos\left(\frac{x}{2}\right) dx.$$

(b) Find the Fourier sine and cosine transform of $e^{-|t|}$.

(4 × 15 = 60 marks)