FOURTH SEMESTER B.TECH. (ENGINEERING) (09 SCHE EXAMINATION, APRIL 2015

EN 09/PTEN 09 401 B-ENGINEERING MATHEMA

(Common for IC, EC, EE, AI, BM, CS and IT)

Time: Three Hours

Maximum: 70 Marks

Part A

Answer all questions.

- 1. Define Gamma distribution.
- 2. Find the z-transform of $x(n) = \left(\frac{1}{2}\right)^n u(-n)$.
- 3. Express x^3 in terms of Legendre polynomials.
- 4. Classify the P.D.E. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.
- 5. Solve the P.D.E. $z = px + qy + \sin(pq)$.

 $(5 \times 2 = 10 \text{ marks})$

Part B

Answer any four questions.

- 1. Two dice are rolled 100 times. Let X be the number of double sixes. Find the probability that exactly 3 times, we get double six. Using (a) Binomial distribution; (b) Poisson distribution.
- 2. X is a uniformly distributed random variable in the interval (-2, 2). Compute:
 - (a) P(X<1).

(b) $P(|X| < \frac{1}{2})$.

(c) P(|X-1|<1).

- (d) Find K such that $P(X > K) = \frac{1}{4}$.
- 3. Using long division, determine the inverse z-transform of $X(z) = \frac{1}{1 \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}}$,

when (a) ROC |z|>1.

Turn over

- 4. Using scaling property, determine the z-transform of:
 - (a) $a^n \cos w_0 n$.

(b) $a^n \sin w_0 n$.

- (c) $2^n u(n-2)$.
- 5. Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.
- 6. Solve $(z^2 2yz y^2)p + (xy + xz)q = xy zx$.

 $(4 \times 5 = 20 \text{ marks})$

Part C

Answer any four questions.

- 1. The mean yield for one-acre plots is 662 kg with a S.D. of 32 kg. Assuming normal distribution, how many one acre plots in a lot of 1000 plots would you expect to have a yield:
 - (a) Over 700 kg.

- (b) Below 600 kg.
- (c) Between 650 and 675 kg.

Or

- 2. (a) If a balanced die is thrown, find the probability that a '6' first appears on the 7th trial.
 - (b) Among the 200 employees of a company, 120 are post graduates. If 8 of the employees are chosen by a lot, find the probability that (i) 4 of the eight will be post graduates; (ii) majority are non-post graduate employees.
- 3. Determine the inverse z-transform of the following X(z) by the partial fraction expansion method:

$$X(z) = \frac{z+2}{2z^2 - 7z + 3}$$
 if the ROC's are:

(a)
$$|z| > 3$$
; (b) $|z| < \frac{1}{2}$; and (c) $\frac{1}{2} < |z| < 3$.

Or

4. Find the inverse z-transform of:

$$\frac{z^3 - 20z}{(z-2)(x^2+4)}$$
 by the residue method.

5. Prove that $(1-2xz+z^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x) z^n$.

6. Prove that (i)
$$J_{\upsilon-1}(x) + J_{\upsilon+1}(x) = \frac{2\upsilon}{x} J_{\upsilon}(x)$$
; (ii) $J_{\upsilon-1}(x) - J_{\upsilon+1}(x) = 2J_{\upsilon}(x)$.

7. Solve the following P.D.E.'s:

(a)
$$p^2q^2 + x^2y^2 = x^2q^2(x^2 + y^2)$$
.

(b)
$$zpq = p + q$$
.

(c)
$$2(p^2-q^2)=3pq$$
.

Or

8. Obtain the D'Alembert's solution of one dimensional wave equation.

 $(4 \times 10 = 40 \text{ marks})$