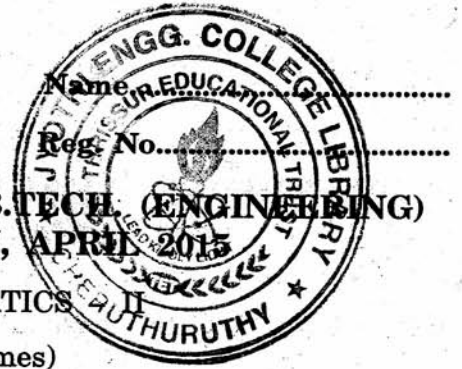


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COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING)
(14 SCHEME) DEGREE EXAMINATION, APRIL 2015

EN 14 102—ENGINEERING MATHEMATICS II

(Common for all B.Tech. Programmes)

Time : Three Hours

Maximum : 100 Marks

Part A

Answer any eight questions.

- I. (a) Solve $(y \cos x + 1)dx + \sin x dy = 0$.
- (b) Solve $\frac{dy}{dx} - x^2 y = y^2 e^{-x^3/3}$.
- (c) Solve the differential equation $(\sqrt{xy} - x)dy + ydx = 0$.
- (d) Find $L^{-1}\left(\log\left(\frac{s+1}{s}\right)\right)$.
- (e) Find $L(t^2 e^{-3t} \sin 2t)$.
- (f) Find $L\left(\frac{e^{at} - \cos bt}{t}\right)$.
- (g) If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$ then prove that $\nabla^2 r^n = n(n+1)r^{n-2}$.
- (h) Find the directional derivative of $u = xy^2 + yz^2$ at the point $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 + 1 = 0$.
- (i) If $\vec{A} = (3x^2 + 6y)i - 14 yzj + 20 xz^2k$, evaluate $\int \vec{A} \cdot d\vec{r}$ from $(0, 0, 0)$ to $(1, 1, 1)$ along the path $x = t, y = t^2, z = t^3$.
- (j) Evaluate $\int_C (\cos x \sin y - xy)dx + \sin x \cos y dy$ by Green's theorem, where C is the circle $x^2 + y^2 = 1$.

(8 × 5 = 40 marks)

Turn over

Part B*Answer all questions.*

II. A (a) Solve the equation $(D^2 + 2D - 3)y = e^x \cos x + x^2$.

(b) Prove that the family of parabolas $x^2 = 4a(y + a)$ is self orthogonal.

Or

B (a) Solve $x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 2y = x^2 + \sin \log x$.

(b) Solve $\frac{d^2 y}{dx^2} + 4y = \sec 2x$ using the methods of variation of parameters.

 $(1 \times 15 = 15 \text{ marks})$

III. A (a) Solve the differential equation :

$$\frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} + 3y = e^{-t}, y(0) = y'(0) = 1.$$

Using Laplace transforms.

(b) Prove that $\int_0^{\infty} x^2 e^{-x^2} dx = \frac{1}{4} \sqrt{\pi}$.

Or

B (a) Apply convolution theorem to evaluate $L^{-1} \left(\frac{1}{(s^2 + a^2)^2} \right)$.

(b) Define unit step function and unit impulse function. Find their Laplace transforms.

 $(1 \times 15 = 15 \text{ marks})$

IV. A (a) Find the value n , if $r^n \bar{r}$ is both solenoidal and irrotational where $\bar{r} = xi + yj + zk$ and $r = |\bar{r}|$.

(b) If $\bar{F} = (y \sin z - \sin x)i + (x \sin z + 2yz)j + (xy \cos z + y^2)k$ then prove that \bar{F} is irrotational and find its scalar potential.

Or

B (a) A particle moves along the curve $x = e^{-t}, y = 2\cos 3t, z = 2\sin 3t$ where t is the time. Determine its velocity and acceleration vectors and also find the components of its velocity and acceleration at $t = 1$ in the direction $i - j + 3k$.

(b) Give the physical interpretation of divergence.

(1 × 15 = 15 marks)

V. A (a) Verify Green's theorem in the plane for $\oint_C (2y - x^2)dx + (x^2 + y^2)dy$ where C is the boundary of the region enclosed by $y = x^2$ and $y^2 = x$.

(b) Use Stoke's theorem to evaluate $\oint_C \bar{F} \cdot d\bar{r}$ where

$\bar{F} = (2x + y - 2z)i + (2x - 4y + z^2)j + (x - 2y + z^2)k$, where C is the circle with centre (0, 0, 3) and radius 5 units in the plane $z = 3$.

Or

B Verify divergence theorem for $\bar{F} = x^2i + zj + yzk$ taken over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

(1 × 15 = 15 marks)

[4 × 15 = 60 marks]