C 80661 (Pages : 3) Name.............................

Reg. No.............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) (14 SCHEME) DEGREE EXAMINATION, APRIL 2015

EN 14 102—ENGINEERING MATHEMATICS II

(Common for all B.Tech. Programmes)

Time : Three Hours Maximum : 100 Marks

Part A

Answer any **eight** *questions.*

I. (a) Solve $(y\cos x + 1)dx + \sin x\, dy = 0$.

(b) Solve $\frac{dy}{dx} - x^2 y = y^2 e^{-x^3/3}$.

(c) Solve the differential equation $\left(\sqrt{xy} - x\right)dy + ydx = 0$.

(d) Find $L^{-1}\left(\log\left(\frac{s+1}{s}\right)\right)$.

(e) Find $L\left(t^2 e^{-3t}\sin 2t\right)$.

(f) Find $L\left(\frac{e^{at} - \cos bt}{t}\right)$.

(g) If $\bar{r} = xi + yj + zk$ and $r = |\bar{r}|$ then prove that $\nabla^2 r^n = n(n+1)r^{n-2}$.

(h) Find the directional derivative of $u = xy^2 + yz^2$ at the point (2, –1, 1) in the direction of the normal to the surface $x \log z - y^2 + 1 = 0$.

(i) If $\bar{A} = \left(3x^2 + 6y\right)i - 14\, yzj + 20\, xz^2 k$, evaluate $\int \bar{A} \cdot d\bar{r}$ from (0, 0, 0) to (1, 1, 1) along the path $x = t, y = t^2, z = t^3$.

(j) Evaluate $\int_C (\cos x \sin y - xy)dx + \sin x \cos y\, dy$ by Green's theorem, where C is the circle $x^2 + y^2 = 1$.

($8 \times 5 = 40$ marks)

Turn over

Part B

Answer **all** *questions.*

II. A (a) Solve the equation $\left(D^2+2D-3\right)y = e^x\cos x + x^2$.

(b) Prove that the family of parabolas $x^2 = 4a(y+a)$ is self orthogonal.

Or

B (a) Solve $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 2y = x^2 + \sin\log x$.

(b) Solve $\frac{d^2y}{dx^2} + 4y = \sec 2x$ using the methods of variation of parameters.

(1 × 15 = 15 marks)

III. A (a) Solve the differential equation :

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = e^{-t}, y(0) = y'(0) = 1.$$

Using Laplace transforms.

(b) Prove that $\int_0^\infty x^2 e^{-x^2}\,dx = \frac{1}{4}\sqrt{\pi}$.

Or

B (a) Apply convolution theorem to evaluate $L^{-1}\left(\frac{1}{\left(s^2+a^2\right)^2}\right)$.

(b) Define unit step function and unit impulse function. Find their Laplace transforms.

(1 × 15 = 15 marks)

IV. A (a) Find the value n, if $r^n\bar{r}$ is both solenoidal and irrotational where $\bar{r} = xi + yj + zk$ and $r = |\bar{r}|$.

(b) If $\bar{F} = (y\sin z - \sin x)i + (x\sin z + 2yz)j + \left(xy\cos z + y^2\right)k$ then prove that $\bar{F}$ is irrotational and find its scalar potential.

Or

B (a) A particle moves along the curve $x = e^{-t}, y = 2\cos 3t, z = 2\sin 3t$ where t is the time. Determine its velocity and acceleration vectors and also find the components of its velocity and acceleration at $t = 1$ in the direction $i - j + 3k$.

(b) Give the physical interpretation of divergence.

(1 × 15 = 15 marks)

V. A (a) Verify Green's theorem in the plane for $\oint_C \left(2y - x^2\right)dx + \left(x^2 + y^2\right)dy$ where C is the boundary of the region enclosed by $y = x^2$ and $y^2 = x$.

(b) Use Stoke's theorem to evaluate $\oint_C \overline{F} \cdot d\overline{r}$ where $\overline{F} = (2x + y - 2z)i + \left(2x - 4y + z^2\right)j + \left(x - 2y + z^2\right)k$, where C is the circle with centre (0, 0, 3) and radius 5 units in the plane $z = 3$.

Or

B Verify divergence theorem for $\overline{F} = x^2 i + zj + yzk$ taken over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

(1 × 15 = 15 marks)

[4 × 15 = 60 marks]