C 80638

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Name.....

Reg. No.....

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) (09 SCHEME) DEGREE [SUPPLEMENTARY] EXAMINATION, APRIL 2015

PTEN/EN 09 101—ENGINEERING MATHEMATICS

Time : Three Hours

Maximum : 70 Marks

Part A

Answer **all** *questions.*

1. Evaluate $\lim_{x \to 0} \frac{a^x - 1}{x}$.

2. Find the Jacobian J if $u = xy^2, v = \frac{x^2}{y}$.

3. Examine the convergence of the series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots \text{to } \infty$.

4. Find the rank of the matrix :

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 6 & 1 & 1 & 1 \\ 1 & 1 & -2 & 0 \end{bmatrix}.$$

5. Express $f(x) = e^x$ as a half range sine series in $0 < x < \pi$.

($5 \times 2 = 10$ marks)

Part B

Answer any **four** *questions.*

6. Find the radius of curvature of $xy = i^2$ at the point $(ct, c/t)$.

7. Test for the convergence of $\sum \frac{2^n n!}{n^n}$.

8. Find the evolute of the parabola $y^2 = 4ax$.

9. Find the internal of convergence of the series $1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \ldots$

Turn over

10. For what values of λ and μ do the system of equations $x+y+z=6, x+2y+3z=10, x+2y+\lambda z=\mu$ have :

(i) No solution. (ii) Unique solution.

(iii) More than one solution.

11. Find the Fourier series expansion for :

$$f(x)=\begin{cases} l-x, & 0\le x\le l \\ 0, & l\le x\le 2l \end{cases}$$

(4 × 5 = 20 marks)

Part C

Answer section (a) **or** *section B of each question.*

12. (a) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{-\sin u\cos 2u}{4\cos^3 u}$.

Or

(b) A rectangular box open at the top, is to have a volume of 32 c.c. Find the dimensions of the box requiring least material for its construction.

13. (a) Discuss the convergence of the series :

$$1+\frac{3}{7}x+\frac{3.6}{7.10}x^2+\frac{3\cdot 6\cdot 9}{7\cdot 10\cdot 13}x^3+\ldots \text{ to } \infty$$

Or

(b) Discuss the convergence of the series :

(i) $\sum_{n=1}^{\infty}\frac{(n+1)^n x^n}{n^{n+1}}$.

(ii) $\frac{x}{2\sqrt{3}}+\frac{x^2}{3\sqrt{4}}+\frac{x^3}{4\sqrt{5}}+\ldots \text{ to } \infty$.

14. (a) Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Hence find A^{-1}.

Or

(b) Reduce the quadratic form $x^2-2xy+y^2+z^2$ to Canonical form by orthogonal transformation. Examine for definiteness. Find the rank, index and signature.

15. (a) Find the Fourier series of :

$f(x) = x + x^2, -\pi < x < \pi$.

Using the series deduce that :

$$\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$$

Or

(b) Expand $f(x) = x \sin x$ as a Fourier series in $0 < x < 2\pi$.

(4 × 10 = 40 marks)