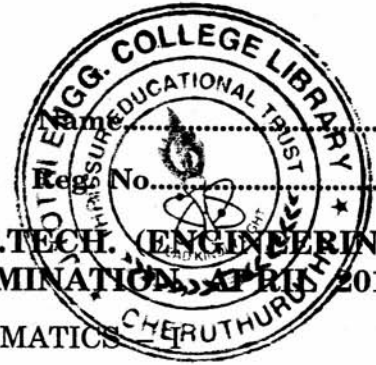


C 80638

(Pages : 3)



**COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING)  
(09 SCHEME) DEGREE [SUPPLEMENTARY] EXAMINATION, APRIL 2015**

PTEN/EN 09 101—ENGINEERING MATHEMATICS

Time : Three Hours

Maximum : 70 Marks

**Part A**

*Answer all questions.*

1. Evaluate  $\lim_{x \rightarrow 0} \frac{a^x - 1}{x}$ .
2. Find the Jacobian J if  $u = xy^2, v = \frac{x^2}{y}$ .
3. Examine the convergence of the series  $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$  to  $\infty$ .
4. Find the rank of the matrix :

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 6 & 1 & 1 & 1 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

5. Express  $f(x) = e^x$  as a half range sine series in  $0 < x < \pi$ .

(5 × 2 = 10 marks)

**Part B**

*Answer any four questions.*

6. Find the radius of curvature of  $xy = i^2$  at the point  $(ct, c/t)$ .
7. Test for the convergence of  $\sum \frac{2^n n!}{n^n}$ .
8. Find the evolute of the parabola  $y^2 = 4ax$ .
9. Find the internal of convergence of the series  $1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \dots$

Turn over

10. For what values of  $\lambda$  and  $\mu$  do the system of equations  $x + y + z = 6$ ,  $x + 2y + 3z = 10$ ,  $x + 2y + \lambda z = \mu$  have :

- (i) No solution. (ii) Unique solution.  
 (iii) More than one solution.

11. Find the Fourier series expansion for :

$$f(x) = \begin{cases} l-x, & 0 \leq x \leq l \\ 0, & l \leq x \leq 2l \end{cases}$$

(4 × 5 = 20 marks)

### Part C

Answer section (a) or section B of each question.

12. (a) If  $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ , prove that  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{-\sin u \cos 2u}{4 \cos^3 u}$ .

Or

(b) A rectangular box open at the top, is to have a volume of 32 c.c. Find the dimensions of the box requiring least material for its construction.

13. (a) Discuss the convergence of the series :

$$1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \dots \text{ to } \infty$$

Or

(b) Discuss the convergence of the series :

(i)  $\sum_{n=1}^{\infty} \frac{(n+1)^n x^n}{n^{n+1}}$ .

(ii)  $\frac{x}{2\sqrt{3}} + \frac{x^2}{3\sqrt{4}} + \frac{x^3}{4\sqrt{5}} + \dots \text{ to } \infty$ .

14. (a) Verify Cayley-Hamilton theorem for the matrix  $A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ . Hence find  $A^{-1}$ .

Or

(b) Reduce the quadratic form  $x^2 - 2xy + y^2 + z^2$  to Canonical form by orthogonal transformation. Examine for definiteness. Find the rank, index and signature.

15. (a) Find the Fourier series of :

$$f(x) = x + x^2, -\pi < x < \pi.$$

Using the series deduce that :

$$\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$

*Or*

(b) Expand  $f(x) = x \sin x$  as a Fourier series in  $0 < x < 2\pi$ .

(4 × 10 = 40 marks)