D 70541 (Pages : 3) Name.................................

Reg. No.................................

FIRST SEMESTER M.TECH. DEGREE EXAMINATION, JANUARY 2015

MCS 10 101—ADVANCED MATHEMATICAL STRUCTURES

Time : Three Hours Maximum : 100 Marks

Answer any **five** *questions, choosing at least* **one** *from each module.*

Module I

I. (a) Establish necessary and sufficient conditions for the wide sense stationary of the process $x(t) = A \cos \omega t + B \sin \omega t$ where A and B are random variables.

(b) Suppose that the customers are arriving at a ticket counter according to a Poisson Process with mean rate of 2 per minute. Then in an interval of 5 minutes, find the probability that the number of customers arriving is :

(i) Exactly 3.

(ii) More than 3.

(iii) Less than 3

(iv) atleast 3

II. (a) Define counting process. If $\{N(t), t \geq 0\}$ is a counting Process, find the probability that there will be exactly n renewals at time t. Explain various type of Renewal Process.

(b) If N (t), $t \geq 0$ is a renewal process with $E(X_n) = \mu$ for all n, then with probability 1, prove that $\lim_{t \to \infty} \frac{N(t)}{t} = \frac{1}{\mu}$.

Module II

III. (a) A message transmission system is found to be Markovian with the transition probability of current message to next message as given by the matrix :

$$P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.7 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}.$$

The initial probabilities of the states are $P_1(0) = 0.4$, $P_2(0) = 0.3$, $P_3(0) = 0.3$.

Find the probabilities of the next message.

Turn over

(b) Consider the Markov Chain with transition probability matrix :

$$\begin{bmatrix} 0.4 & 0.6 & 0 & 0 \\ 0.3 & 0.7 & 0 & 0 \\ 0.2 & 0.4 & 0.1 & 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Is it irreducible ? If No, find the classes. Find the nature of states.

IV. (a) Define the following :

(i) Discrete time Markov Chain.

(ii) Semi Markov Process.

(iii) Continuous Markov Chain.

(iv) Irreducible Chain.

(b) (i) Define birth process and pure birth process. For a pure birth process, what are $P_1(0)$ and $P_2(0)$.

(ii) Define a birth and death process. For a birth and death process what is $P_{-1}(t)$?

Module III

V. (a) The capacity of a communication line is 2000 bits per second. This line is used to transmit eight–bit characters, so the maximum rate is 250 characters per second. The application calls for traffic from many devices to be sent on the line with a total volume of 12,000 characters per minute. Determine :

(i) The line utilization.

(ii) The average number of characters waiting to be transmitted ?

(b) Consider the problem of designing a system with two identical processors. We have two independent job streams with respective average arrival rates 20 / hr and 15 / hr. The average service time for both job types is 2 minutes. Should we dedicate a processor per job stream or should we pool the job streams and processors together ?

VI. (a) Explain the method of monitoring the network traffic.

(b) Explain how forecasting the network traffic is done using Queuing Theory.

Module IV

VII. (a) A group of engineers has 2 terminals available to aid in their calculations. The average computing job requires 20 minutes of terminal time and each engineer requires some computations about once every half an hour. Assume that these are distributed according to on exponential distribution. If there are 6 engineers in the group, find the total time lost per day.

(b) A petrol pump station has 4 pumps. The service times follow the exponential distribution with a mean of 6 minutes and cars arrive for service in a Poisson process at the rate of 30 cars per hour.

(i) Find the average waiting time in the queue, the average time spent in the system; and

(ii) For what percentage of time would a pump be idle on an average ?

VIII. (a) Consider the closed queueing network. There are three customers who are doomed forever to cycle between queue 1 and queue 2. The service times at the queues are independent and exponentially distributed with mean μ_1 and μ_2. Assume that $\mu_2 < \mu_1$.

(i) The system can be represented by a four state Markov Chain. Find the transition rate of the chain ?

(ii) Find the stéady state probabilities of the state ?

(iii) Find the customer arrival rate at queue 1.

(iv) Find the rate at which a customer cycles through the system.

(b) If people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 minutes before the picture starts and if its take exactly 1.5 minute to reach the correct seat after purchasing the ticket.

(i) Can he expect to be seated for the start of the picture.

(ii) How early must be arrive in order to be 99% sure of being seated for the start of the picture.

($5 \times 20 = 100$ marks)