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Name.............................

Reg. No..........................

FIRST SEMESTER M.TECH. DEGREE EXAMINATION, JANUARY 2015

EPD/EPE/EPS 10 102—SYSTEM DYNAMICS

Time : Three Hours Maximum : 100 Marks

Answer any **five** *questions choosing at least* **one** *from each module.*

Module I

1. Obtain a state space representation of the system described by the equation :

   (a) $Y(K+Z) + Y(K+1) + 0.16\, Y(K) = u(K+1) + 2u(K).$ (10 marks)

   (b) $\dfrac{Y(Z)}{u(z)} = \dfrac{z^{-1} + 2z^{-2}}{1 + 4z^{-4} + 3z^{-2}}.$ (10 marks)

2. A discrete time system is described by the difference equation :

   $Y(K+2) + 5Y(K+1) + 6\, Y(K) = u(K)$

   $Y(0) = Y(1) = 0$; $T = 1$ sec.

   (a) Determine a state model in canonical form. (8 marks)

   (b) Find the state transition matrix. (5 marks)

   (c) For input $u(K) = 1,\ K \geq 1,$ find the output $Y(K)$. (7 marks)

Module II

3. Determine the stability of the origin of the discrete-time system :

$$\begin{bmatrix} X_1(K+1) \\ X_2(K+1) \\ X_3(K+1) \end{bmatrix} = \begin{bmatrix} 1 & 3 & 0 \\ -3 & -2 & -3 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1(K) \\ X_2(K) \\ X_3(K) \end{bmatrix}.$$

(20 marks)

4. (a) With an example, explain the stability analysis of non-linear system. (10 marks)

   (b) Elucidate the function of variable gradient method with an example. (10 marks)

Turn over

Module III

5. Determine the state controllability for the systems represented by the following state equations :

(a) $\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u.$ (10 marks)

(b) $\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u.$ (10 marks)

6. The control system defined by :

$$\begin{bmatrix} X_1 (K+1) \\ X_2 (K+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix} \begin{bmatrix} X_1 (K) \\ X_2 (K) \end{bmatrix} + \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} u(K)$$

$$\begin{bmatrix} X_1 (0) \\ X_2 (0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

is completely state controllable. Determine a sequence of control signals u (0) and u (1) such that the state x (2) becomes :

$$\begin{bmatrix} X_1 (2) \\ X_2 (2) \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

(20 marks)

Module IV

7. With an example, explain the procedure for solution of reduced Riccatti equation. (20 marks)

8. Consider the system :

$$X (K + 1) = G\, x (K) + H\, u(K)$$

where $G = \begin{bmatrix} 0 & 0 \\ -0.5 & 1 \end{bmatrix}, H = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, x(0) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

and the performance index

$$J = \frac{1}{2} \sum_{K=0}^{\infty} \left[X^*(k)\, Q\, x(k) + u^*(k) R\, u(k) \right]$$

where

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}, R = 1.$$

Determine the optimal control law to minimize the performance index. Also determine the minimum value of J.

(20 marks)

[5 × 20 = 100 marks]