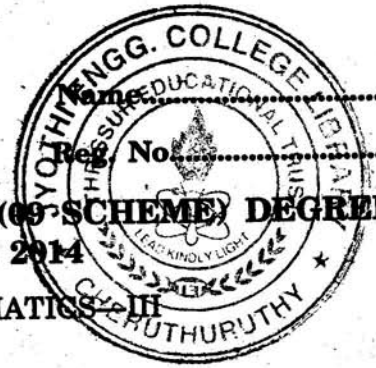


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**THIRD SEMESTER B.TECH. (ENGINEERING) (09 SCHEME) DEGREE  
EXAMINATION, NOVEMBER 2014**

**EN 09 301—ENGINEERING MATHEMATICS—III**

(Common to all Branches)

Time : Three Hours

Maximum : 70 Marks

**Part A**

*Answer all questions.*

1. Determine whether the Cauchy-Riemann conditions are satisfied for  $w = e^{-z}$ .
2. Define conformal mapping.
3. Find the residue of  $\frac{\sin z}{z}$  at its singularity.
4. How do you define linear independence of a set of vectors in a vector space?
5. Find the inverse Fourier transform of  $\frac{1}{iw + 5}$ .

(5 × 2 = 10 marks)

**Part B**

*Answer any four questions.*

6. Show that  $e^x (x \cos y - y \sin y)$  is a harmonic function. Find the analytic function for which  $e^x (x \cos y - y \sin y)$  is the imaginary part.
7. Find the image of the line  $x + y = 2$  under the transformation  $w = z^2$ .
8. Evaluate  $\int_C \frac{dz}{(z^2 + 4)^2}$  where C is the circle  $|z - i| = 2$ .
9. Find a basis, the dimension of the subspace W of  $\mathbb{R}^4$  generated by  $(1, -4, -2, 1)$ ,  $(1, -3, -1, 2)$  and  $(3, -8, -2, 7)$ .
10. Verify Schwartz's inequality for the vectors  $x = (1 + i, -2 - 2i, -5i)$  and  $y = (-3 + 2i, 2, 4 - 4i)$  in  $\mathbb{C}^3$ .

Turn over

11. Find the Fourier integral representation of the function  $f(t) = \begin{cases} 0, & t < 0 \\ e^{-t}, & t \geq 0 \end{cases}$ .

Hence evaluate  $\int_0^{\infty} \frac{1}{1+w^2} dw$ .

(4 × 5 = 20 marks)

**Part C**

*Answer all questions as per choice given.*

12. (a) If  $u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}$  find  $f(z) = u - iv$  which is analytic. Given that  $f(\pi/2) = 1$ .

*Or*

- (b) Find the bilinear transformation which maps the points  $z = -2i, i, \infty$  onto the points  $w = 0, -3, \frac{1}{3}$  respectively. Find the image of  $|z| < 1$ .

13. (a) Find the Taylor's or Laurent's series expansion of the function  $f(z) = \frac{7z-2}{z(z+1)(z-2)}$  in

(i)  $|z| < 1$ .

(ii)  $1 < |z+1| < 3$ .

(iii)  $|z+1| > 3$ .

*Or*

- (b) Evaluate  $\int_C \frac{2z^2 - 1}{z^2(z+1)^2(2z+1)} dz$  where C is the circle  $|z| = 1.5$ .

14. (a) Find the co-ordinates of the vectors  $\{(2, -5, 2), (-7, 5, 9), (8, -3, -4)\}$  relative to the basis  $S = \{(1, 2, 1), (2, 1, 0), (1, -1, 2)\}$  of  $\mathbb{R}^3$ .

*Or*

- (b) Show that the polynomials  $P_1(x) = -1 + 2x + x^2$ ,  $P_2(x) = 2 + x$ ,  $P_3(x) = x + x^2$  form a basis for  $P_2(x)$ . Use Gram-Schmidt process to generate an orthonormal basis from this basis using the

$$\text{innerproduct } \langle f, g \rangle = \int_0^1 f(x) g(x) dx.$$

15. (a) Find the Fourier sine and cosine transform of the function  $f(t)$  defined by

$$f(t) = \begin{cases} t & , 0 < t < 1 \\ 2-t & , 1 < t < 2. \\ 0 & , t \geq 2 \end{cases}$$

Or

- (b) Find the Fourier transform of  $f(t) = \begin{cases} 1-t^2, & |t| < 1 \\ 0 & , |t| > 1 \end{cases}$ .

$$\text{Hence evaluate } \int_0^{\infty} \left( \frac{x \cos x - \sin x}{x^3} \right)^2 dx.$$

(4 × 10 = 40 marks)