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Name.................................

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THIRD SEMESTER B.TECH. (ENGINEERING) (09 SCHEME) DEGREE EXAMINATION, NOVEMBER 2014

EN 09 301—ENGINEERING MATHEMATICS—III

(Common to all Branches)

Time : Three Hours Maximum : 70 Marks

Part A

Answer **all** *questions.*

1. Determine whether the Cauchy-Riemann conditions are satisfied for $w = e^{-z}$.

2. Define conformal mapping.

3. Find the residue of $\dfrac{\sin z}{z}$ at its singularity.

4. How do you define linear independence of a set of vectors in a vector space ?

5. Find the inverse Fourier transform of $\dfrac{1}{iw + 5}$.

(5 × 2 = 10 marks)

Part B

Answer any **four** *questions.*

6. Show that $e^x (x \cos y - y \sin y)$ is a harmonic function. Find the analytic function for which $e^x (x \cos y - y \sin y)$ is the imaginary part.

7. Find the image of the line $x + y = 2$ under the transformation $w = z^2$.

8. Evaluate $\int_C \dfrac{dz}{(z^2 + 4)^2}$ where C is the circle $|z - i| = 2$.

9. Find a basis, the dimension of the subspace W of R^4 generated by (1, – 4, – 2, 1), (1, – 3, – 1, 2) and (3, – 8, – 2, 7).

10. Verify Schwartz's inequality for the vectors $x = (1 + i, -2 - 2i, -5i)$ and $y = (-3 + 2i, 2, 4 - 4i)$ in C^3.

Turn over

11. Find the Fourier integral representation of the function $f(t) = \begin{cases} 0, & t < 0 \\ e^{-t}, & t \geq 0 \end{cases}$.

Hence evaluate $\int_0^\infty \frac{1}{1+w^2} dw$.

(4 × 5 = 20 marks)

Part C

Answer **all** *questions as per choice given.*

12. (a) If $u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$ find $f(z) = u - iv$ which is analytic. Given that $f(\pi/2) = 1$.

Or

(b) Find the bilinear transformation which maps the points $z = -2i, i, \infty$ onto the points $w = 0, -3, \frac{1}{3}$ respectively. Find the image of $|z| < 1$.

13. (a) Find the Taylor's or Laurent's series expansion of the function $f(z) = \frac{7z-2}{z(z+1)(z-2)}$ in

(i) $|z| < 1$.

(ii) $1 < |z+1| < 3$.

(iii) $|z+1| > 3$.

Or

(b) Evaluate $\int_C \frac{2z^2 - 1}{z^2 (z+1)^2 (2z+1)} dz$ where C is the circle $|z| = 1.5$.

14. (a) Find the co-ordinates of the vectors { (2, – 5, 2), (– 7, 5, 9), (8, – 3, – 4)} relative to the basis S = {(1, 2, 1), (2, 1, 0), (1, – 1, 2)} of $\mathbb{R}^3$.

Or

(b) Show that the polynomials $P_1(x) = -1 + 2x + x^2$, $P_2(x) = 2 + x$, $P_3(x) = x + x^2$ form a basis for $P_2(x)$. Use Gram-Schmidt process to generate an orthonormal basis from this basis using the innerproduct $\langle f, g \rangle = \int_0^1 f(x)\, g(x)\, dx$.

15. (a) Find the Fourier sine and cosine transform of the function $f(t)$ defined by

$$f(t) = \begin{cases} t & , 0 < t < 1 \\ 2 - t, & 1 < t < 2. \\ 0 & , t \geq 2 \end{cases}$$

Or

(b) Find the Fourier transform of $f(t) = \begin{cases} 1 - t^2, & |t| < 1 \\ 0 \quad , & |t| > 1 \end{cases}$.

Hence evaluate $\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3} \right)^2 dx$.

(4 × 10 = 40 marks)