C 60696 **(Pages : 2)** Name.....

Reg. No.....

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, APRIL 2014

(2009 Scheme)

PTEN / EN 09 101—ENGINEERING MATHEMATICS—I

Time : Three Hours Maximum : 70 Marks

Part A

Answer **all** *questions.*
Each question carries 2 *marks.*

1. Find the radius of curvature at the point (c, c) on the curve $xy = c^2$.

2. State Comparison Test.

3. Find the sum and product of the eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & -3 \end{bmatrix}$.

4. Find the index and signature of the quadratic factor ${x_1}^2 + 2{x_2}^2 - 3{x_3}^2$.

5. Find the Fourier constant b_n for $x \sin x$ in $(-\pi, \pi)$.

(5 × 2 = 10 marks)

Part B

Answer any **four** *questions.*
Each question carries 5 *marks.*

6. If z is a homogeneous function of degree n in x and y, show that

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial z^2} = n(n-1)z.$$

7. Test for convergence the series $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \ldots + \infty$.

8. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$.

9. Determine the rank of the following matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$.

Turn over

10. Find the Fourier series of the function $f(x) = 2x - x^2$ for $0 < x < 3$ and $f(x+3) = f(x)$.

11. Find the half range cosine series for the function $f(x) = x^2$ in the range $0 \le x \le \pi$.

(4 × 5 = 20 marks)

Part C

Answer Section (a) **or** *Section (b) of each question.*
Each question carries 10 *marks.*

12. (a) If ρ is the radius of curvature at any point (x, y) on the curve $y = \frac{ax}{a+x}$. Show that

$$\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2.$$

Or

(b) Find the evolute of the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

13. (a) Discuss the convergence of the series $1 + \frac{(1!)^2}{2!}x + \frac{(2!)^2}{4!}x^2 + \frac{(3!)^2}{6!}x^3 + \ldots$

Or

(b) Expand $\log_e x$ in powers of $(x-1)$ and hence evaluate $\log_e 1.1$ correct to 4 decimal places.

14. (a) Reduce the quadratic form $2x_1^2 + 6x_2^2 + 2x_3^2 + 8x_1x_3$ to Canonical form by orthogonal reduction. Find also the nature of the quadratic form.

Or

(b) Test for consistency and solve $5x + 3y + 7z = 4$; $3x + 26y + 2z = 9$; $7x + 2y + 10z = 5$.

15. (a) Find the Fourier Series expansion of period $2l$ for the function $f(x) = (l-x)^2$ in the range $(0, 2l)$. Deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \pi^2/6$.

Or

(b) Find the half range cosine series for $f(x) = \begin{cases} x^2 & , 0 < x < 1 \\ 2 - x & , 1 < x < 2 \end{cases}$.

(4 × 10 = 40 marks)