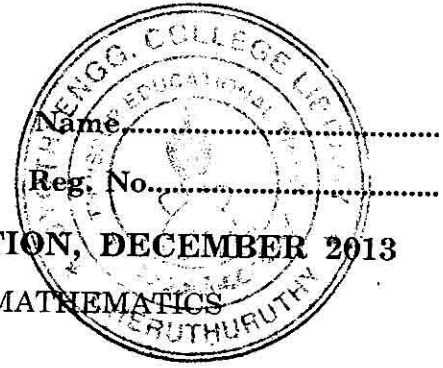


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FIRST SEMESTER M.TECH. DEGREE EXAMINATION, DECEMBER 2013

EIC 11 101/EPD/EPE/EPS 10 .101—APPLIED MATHEMATICS

(2010 Scheme)

Time : Three Hours

Maximum : 100 Marks

Answer any five questions by choosing at least one question from each module.

Each question carried 20 marks.

Module I

- I. (a) Out of 800 families with 4 children each, how many families would be expected to have
- (i) 2 boys and 2 girls.
 - (ii) at least 1 boy.
 - (iii) atmost 2 girls and
 - (iv) children of both sexes. Assume equal probabilities for boys and girls.
- (b) In a given city, 4% of all licenced drivers will be involved in atleast 1 road accident in any given year. Determine the probability that among 150 licenced drivers randomly chosen in this city :
- (i) only 5 will be involved in at least 1 accident in any given year and
 - (ii) atmost 3 will be involved in at least 1 accident in any given year.
 - (iii) atleast 3 will be involved in at least 1 accident in any given year.

(20 marks)

- II. (a) A sample of 12 measurements of the breaking strengths of cotton threads gave a mean of 7.38 oz and a standard deviation of 1.24 oz. Find

- (i) 95 %.
- (ii) 99%.
- (iii) 90%.

Confidence limits for the actual mean breaking strength.

Turn over

- (b) Measurements of a sample of weights were determined as 8.3, 10.6, 9.7, 8.8, 10.2 and 9.4 lb respectively. Determine unbiased and efficient estimates of:
- the population mean.
 - the population variance.
 - compare the sample standard deviation with the estimated population standard deviation.
- (c) What are the steps in testing a statistical hypothesis ?

(20 marks)

Module II

- III. (a) A chemical company wishing to study the effect of extraction time on the efficiency of an extraction operation, obtained the data shown in the following table :

Extraction time									
(min) (x)	:	27	45	41	19	35	39	19	49
Extraction									
efficiency (%) (y)	:	57	64	80	46	42	72	52	77

Obtain the two lines of regression and the correlation coefficient. Predict the extraction efficiency one can expect when the extraction time is 35 minutes.

- (b) The following are data on the number of twists required to break a certain kind of forged alloy bar and the percentages of two alloying elements present in the metal.

No. of twists	:	41	49	69	65	40	50	58	57	31
% of element A x_1	:	1	2	3	4	1	2	3	4	1
% of element B x_2	:	5	5	5	5	10	10	10	10	15

Fit a least squares regression plane and use it to estimate the number of twists required to break one of the bars when $x_1 = 2.5$ and $x_2 = 12$.

(20 marks)

IV. (a) Given the following observations collected according to the one-way analysis of variance design.

Treatment 1	:	6	4	5		
Treatment 2	:	13	10	13	12	
Treatment 3	:	7	9	11		
Treatment 4	:	3	6	1	4	1

Construct the analysis of variance table and test the equality of treatments using $\alpha = 0.05$.

(b) Explain the basic principles of experimental design.

(20 marks)

Module III

V. (a) If $U(t) = X \cos t + Y \sin t$ and $V(t) = Y \cos t + X \sin t$ where X and Y are independent random variables such that $E(X) = 0$, $E(Y) = 0$, $E(X^2) = E(Y^2) = 1$, show that $\{U(t)\}$ and $\{V(t)\}$ are individually stationary in the wide sense, but not jointly stationary.

(b) Define a stochastic process. What are the different types of stochastic process. Explain the examples. Define a stationary process and show that the mean and variance of a first order stationary process are continuous.

(20 marks)

VI. (a) Three Children (denoted by 1, 2, 3) arranged in a circle play a game of throwing a ball to one another. At each stage the child having the ball is equally likely to throw it into any one of the other two children. Suppose that X_n denotes the child who had the ball after n throws, show X_n forms a Markov Chain. Find the tmp P . Also calculate :

(i) $P\{X_2 = 1 \mid X_0 = 1\}$.

(ii) $P\{X_3 = 2 \mid X_0 = 3\}$.

(iii) $P\{X_3 = 3 \mid X_0 = 2\}$.

(iv) The probability that the child who had originally the ball will have it after two throws.

(v) Find P if the number of children is $m \geq 3$.

(b) Explain Markov-Bernoulli Chain.

(20 marks)

Turn over

Module IV

- VII. (a) If the time to failure T follows an exponential distribution, then give its p.d.f. Find the Reliability $R(t)$, Hazard rate $z(t)$, mean time to failure (MTTF), $\text{Var}(T)$, and the conditional reliability $R(t/t_0)$.
- (b) The early failure rate of a component is given by $z(t) = ae^{-bt}$. Determine the probability of survival of the component from age T for a mission time t hours, given that the component has survived upto age T .

(20 marks)

VIII. Define the following terms :

- (a) Reliability measure.
- (b) Failure rate.
- (c) Flat tree diagram.
- (d) Unidiability.
- (e) Weibull distribution.

(20 marks)