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FIRST SEMESTER M.TECH. DEGREE EXAMINATION, DECEMBER 2013

EIC 11 101/EPD/EPE/EPS 10 .101—APPLIED MATHEMATICS

(2010 Scheme)

Time : Three Hours Maximum : 100 Marks

Answer any **five** *questions by choosing at least* **one** *question from each module.*

Each question carried 20 marks.

Module I

I. (a) Out of 800 families with 4 children each, how many families would be expected to have

(i) 2 boys and 2 girls.

(ii) at least 1 boy.

(iii) atmost 2 girls and

(iv) children of both sexes. Assume equal probabilities for boys and girls.

(b) In a given city, 4% of all licenced drivers will be involved in atleast 1 road accident in any given year. Determine the probability that among 150 licenced drivers randomly chosen in this city :

(i) only 5 will be involved in at least 1 accident in any given year and

(ii) atmost 3 will be involved in at least 1 accident in any given year.

(iii) atleast 3 will be involved in at least 1 accident in any given year.

(20 marks)

II. (a) A sample of 12 measurements of the breaking stregths of cotton threads gave a mean of 7.38 oz and a standard deviation of 1.24 oz. Find

(i) 95 %. (ii) 99%.

(iii) 90%.

Confidence limits for the actual mean breaking strength.

Turn over

(b) Measurements of a sample of weights were determined as 8.3, 10.6, 9.7, 8.8, 10.2 and 9.4 lb respectively. Determine unbiased and efficient estimates of :

(i) the population mean.

(ii) the population variance.

(iii) compare the sample standard deviation with the estimated population standard deviation.

(c) What are the steps in testing a statistical hypothesis ?

(20 marks)

Module II

III. (a) A chemical company wishing to study the effect of extraction time on the efficienty of an extraction operation, obtained the data shown in the following table :

Extraction time (min) (x)	:	27	45	41	19	35	39	19	49
Extraction efficiency (%) (y)	:	57	64	80	46	42	72	52	77

Obtain the two lines of regression and the correlation coefficient. Predict the extraction efficiency one can expect when the extraction time is 35 minutes.

(b) The following are data on the number of twists required to break a certain kind of forged alloy bar and the percentages of two alloying elements present in the metal.

No. of twists	:	41	49	69	65	40	50	58	57	31
% of element A x_1	:	1	2	3	4	1	2	3	4	1
% of element B x_2	:	5	5	5	5	10	10	10	10	15

Fit a least squares regression plane and use it to estimate the number of twists required to break one of the bars when $x_1 = 2.5$ and $x_2 = 12$.

(20 marks)

IV. (a) Given the following observations collected according to the one-way analysis of variance design.

Treatment 1	:	6	4	5		
Treatment 2	:	13	10	.13	12	
Treatment 3	:	7	9	11		
Treatment 4	:	3	6	1	4	1

Construct the analysis of variance table and test the equality of treatments using $\alpha = 0.05$.

(b) Explain the basic principles of experimental design.

(20 marks)

Module III

V. (a) If $U(t) = X\cos t + Y\sin t$ and $V(t) = Y\cos t + X\sin t$ where X and Y are independent random variables such that $E(X) = 0$, $E(Y) = 0$, $E(X^2) = E(Y^2) = 1$, show that $\{U(t)\}$ and $\{V(t)\}$ are individually stationary in the wide sense, but not jointly stationary.

(b) Define a stochastic process. What are the different types of stochastic process. Explain the examples. Define a stationary process and show that the mean and variance of a first order stationary process are continuous.

(20 marks)

VI. (a) Three Children (denoted by 1, 2, 3) arranged in a circle play a game of throwing a ball to one another. At each stage the child having the ball is equally likely to throw it into any one of the other two children. Suppose that X. denotes the child who had the ball after n throws, show X_n forms a Markov Chain. Find the tmp P. Also calculate :

(i) $P\{X_2 = 1 \mid X_0 = 1\}$.

(ii) $P\{X_3 = 2 \mid X_0 = 3\}$.

(iii) $P\{X_3 = 3 \mid X_0 = 2\}$.

(iv) The probability that the child who had originally the ball will have it after two throws.

(v) Find P if the number of children is $m \geq 3$.

(b) Explain Markov-Bernoulli Chain.

(20 marks)

Turn over

Module IV

VII. (a) If the time to failure T follows an exponential distribution, then give its p.d.f. Find the Reliability $R(t)$, Hazard rat $z(t)$, mean time to failure (MTTF), Var (T), and the conditional reliability $R(t/t_0)$.

(b) The early failure rate of a component is given by $z(t) = ae^{-bt}$. Determine the probability of survival of the component from age T for a mission time t hours, given that the component has survived upto age T.

(20 marks)

VIII. Define the following terms :

(a) Reliability measure.

(b) Failure rate.

(c) Flat tree diagram.

(d) Unidiability.

(e) Weibull distribution.

(20 marks)