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FIRST SEMESTER M.TECH. DEGREE EXAMINATION, DECEMBER 2013

Computer Science and Engineering

MCS 10 101—ADVANCED MATHEMATICAL STRUCTURES

Time : Three Hours Maximum : 100 Marks

Answer any **five** *questions, choosing atleast* **one** *question from each module.*
Each question carries 20 *marks.*

Module I

I. (a) Define a Poisson process. Show that the sum of two independent Poisson processes is a Poisson process ; but the difference of two independent Poisson processes is not a Poisson Process.

(b) If $\{X(t)\}$ is a Poisson process, prove that $P\left(X(s)=m/X(t)=n\right)=n\,C_m\left(\frac{s}{t}\right)^m\left(1-\frac{s}{t}\right)^{n-m}$ where $s<t$.

II. (a) Define Renewal Process. Prove that the renewal function of the Poisson process $=\lambda t$.

(b) Prove that the integral equation satisfied by the renewal function M (t), called the renewal equation is given by $M(t)=F(t)+\int_0^t m(t-x)\,f(x)\,dx$.

Module II

III. (a) Consider the Markov chain with transition probability matrix.

$$\begin{bmatrix} 0.4 & 0.6 & 0 & 0 \\ 0.3 & 0.7 & 0 & 0 \\ 0.2 & 0.4 & 0.1 & 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

is it irreducible. If not, find the classes. Find the nature of states.

(b) For a cascade of binary communication channels let $P(X_0=1)=a,\ P(X_0=0)=1-a,\ a>0$. Compute the probability that a one was transmitted given that a one was received after the n^{th} stage.

IV. (a) Define Birth and Death process. Obtain the equations :

(i) $P_n'(t) = -(\lambda_n + \mu_n) P_n(t) + \lambda_{n-1} P_{n-1}(t) + \lambda_{n+1} P_{n+1}(t), n \geq 1.$

(ii) $P_0'(t) = -\lambda_0 P_0(t) + \mu_1 P_1(t)$ of Birth and Death process.

(b) Write short notes on :

(i) Discrete time Markov chains.

(ii) Continuous time Markov chains.

(iii) Semi Markov chains.

Module III

V. (a) Arrivals at a telephone booth are considered to be a Poisson with an average time of 12 min. between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 min.

(i) Find the average number of persons waiting in the system.

(ii) What is the probability that a person arriving at the booth will have to wait in the queue ?

(iii) What is the probability that it will take him more than 10 min. altogether to wait for the phone and complete his call ?

(iv) Estimate the fraction of the day when the phone will be in use.

(v) The telephone department will install a second booth, when convinced that an arrival has to wait on the average for atleast 3 min for phone. By how much the flow of arrivals should increase in order to justify a second booth ?

(vi) What is the average length of the queue that forms from time to time ?

(b) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letter per hour.

(i) What fraction of the time all the typists will be busy ?

(ii) What is the average number of letters waiting to be typed ?

(iii) What is the average time a letter has to spend for waiting and for being typed ?

(iv) What is the probability that a letter will take longer than 20 min waiting to be typed and being typed ?

VI. (a) A petrol pump station has 4 pumps. The service times follow the exponential distribution with a mean of 6 min and cars arrive for service in a Poisson process at the rate of 30 cars per hour.

(i) What is the probability that an arrival would have to wait in line ?

(ii) Find the average waiting time, average time spent in the system and the average number of cars in the system.

(iii) For what percentage of time would a pump be idle on an average ?

(b) Obtain the formulae for the :

(i) Average number of customers in the system.

(ii) Average number of customers in the queue.

(iii) Expected number of customers in non-empty queues.

(iv) The probability that the number of customers in the system greater than or equal to K.

(v) Average waiting time of a customer in the queue for a $(M / M / I) : (\infty / FIFO)$ queue model.

Module IV

VII. (a) A PABX provides queuing and automatic call back facility for outgoing calls. If there are 20 outgoing calls requests per hour and if the average call duration is three minutes, how many trunks are needed to ensure delays less than 30 minutes for 90 percent of the requests ?

(b) Consider an $N \times N$ variable-length packet switch and its closed-queuing network model. Assume i.i.d. exponential packet length with unit mean and independent uniform routing. Using the product from expression for this model, show that saturation throughout is given by $\frac{N-1}{2N-1}$. Observe that as $N \to \infty$ the throughput is 0.5.

VIII. (a) A person enters a bank and find all of the four clerks busy serving customers. There are no other customers in the bank, so the person will stard service as soon as one of the customers in service leaves customers have independent identical, exponential distribution of service time.

(i) What is the probability that the person will be the last to leave the bank assuming that no other customers arrive ?

(ii) If the average service time is 1 minute, what is the average time the person will spend in the bank ?

(iii) Will the answer in part (a) change if there are some additional customers waiting in a common queue and customers begin service in the order of their arrival.

(b) Write a short notes on :

(i) Types of blocking.

(ii) Aggregating Markovian Status.