C 47943 (Pages 3) Name..............................

Reg. No..............................

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2008

EN 04 301 B—MATHEMATICS—III

(For CS and IT)

[2004 admissions]

Time : Three Hours Maximum : 100 Marks

Answer **all** *the questions.*

1. (a) Define a linear transformation. Is the mapping from R^2 to R^2 defined by $T(x_1, x_2) = (x_1 - x_2, 0)$ linear ?

   (b) For what value of k will the vector $u = (1, -2, k)$ be a linear combination of the vectors $v = (3, 0, -2)$ and $w = (2, -1, -5)$

   (c) Find the Fourier integral representation of the function :

   $$f(x) = 0, x < 0$$
   $$= 1, 0 \le x \le 1$$
   $$= 0, x > 1$$

   Hence show that $\int_0^\infty \frac{\sin(x/2)}{x}\, dx = \frac{\pi}{2}$.

   (d) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$.

   (e) Prove that the function $\sinh z$ is analytic and find its derivative.

   (f) Show that the image of the hyperbola $x^2 - y^2 = 1$ under the transformation $w = \frac{1}{z}$ is the lemniscate $\rho^2 = \cos(2\phi)$ where $\rho = e^{i\phi}$.

   (g) Show that $\oint_C \frac{dz}{\left(z^2+4\right)^2} = \frac{\pi}{16}$, $C : |z - i| = 2$.

   (h) Obtain the Larurent's expansion of the function $f(z) = \frac{1}{z^2 \sinh z}$ and evaluate $\int_C \frac{dz}{z^2 \sinh z}$ where C is the circle $|z - 1| = 2$.

(8 × 5 = 40 marks)

Turn over

2. (a) (i) Let U and W subspace of R^4 defined by :

$$U = \{(a, b, c, d) : b + c + d = 0\},\ W = \{(a, b, c, d) : a + b = 0,\ c = 2d\}.$$

Find the dimension and a basis of (i) U ; (ii) $U \cap W$.

(8 marks)

(ii) Find T (a, b) were $T : R^2 \rightarrow R^3$ is defined by $T(1, 2) = (3, -1, 5)$ and $T(0, 1) = (2, 1, -1)$.

(7 marks)

Or

(b) (i) Use Gram-Schemidt process to get an orthonormal basis for the basis (1, 2, 2), (2, 1, – 2), (2, – 2, 1).

(8 marks)

(ii) Let V and W are vector spaces. If $T : V \rightarrow W$ is an invertible linear transformation, then prove that its inverse $T^{-1} : W \rightarrow V$ is linear.

(7 marks)

3. (a) Find the Fouier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$

Hence evaluate $\int_0^\infty \frac{x \cos x - \sin x}{x^3} \cos\left(\frac{x}{2}\right) dx.$

(15 marks)

Or

(b) Find the Fourier transform of $f(x) = \frac{1}{1 + x^2}$. Hence derive the Fourier transform of $\phi(x) = \frac{x}{1 + x^2}$.

(15 marks)

4. (a) (i) Find the analytic function $f(z) = U + iV$ if $U - V = (x - y)(x^2 + 4xy + y^2)$. (8 marks)

(ii) Find the bilinear transformation which maps the points $z = i, -i, 1$ into $0, 1, \infty$ respectively.

(7 marks)

Or

(b) (i) Find the analytic function whose real part is $e^x [(x^2 - y^2) \cos y - 2xy \sin y]$. (8 marks)

(ii) Show that the transformation $w = i \frac{(1 - z)}{(1 + z)}$ transforms the circle $|z| = 1$ into real axis of w-plane and the interior of the circle $|z| < 1$ into the upper half of the w-plane.

(7 marks)

5. (a) (i) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta} d\theta$. (8 marks)

(ii) If $f(a) = \int_C \frac{3z^2 + 7z + 1}{z - a} dx$, where C is the circle $x^2 + y^2 = 4$, find the values of $f(3)$, $f'(1 - i)$, $f''(1 - i)$.

(7 marks)

Or

(b) (i) Evaluate $\int_0^{2\pi} \frac{d\theta}{1 - 2a\sin\theta + a^2}, 0 < a < 1$. (8 marks)

(ii) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$, where C is $|z| = 3$. (7 marks)

[4 × 15 = 60 marks]