C 59093 (Pages : 3) Name..............................

Reg. No..............................

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2009

EN 04 301— (A) ENGINEERING MATHEMATICS

(2004 Admissions)

[Common to all branches except CS and IT]

Time : Three Hours Maximum : 100 Marks

Part A

1. (a) Determine whether u and v are linearly dependent where :

   (i) $u = (3, 4), v = (1, -3)$; (ii) $u = (2, -3), v = (6, -9)$.

   (b) Determine whether or not the vectors (1, –2, 1), (2, 1, –7), (7, –4, 1) are linearly dependent.

   (c) Express $f(x) = \begin{cases} 1 \text{ for } |x| \le 1 \\ 0 \text{ for } |x| > 1 \end{cases}$ as a Fourier integral.

   (d) Find the Fourier transform (Complex) of $f(x) = e^{ikx}, a < x < b$

   $= 0, x < a \text{ and } x > b$.

   (e) In 256 sets of 12 tosses of a coin, in how many cases one can expect 8 heads and 4 tails.

   (f) If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2,000 individuals more than two will get a bad reaction.

   (g) Write any two points about Number of degrees of freedom ?

   (h) An unbiased coin is thrown "n" times. H is desired that the relative frequency of the appearance of heads should lie between 0.49 and 0.51. Find the smallest value of n that will ensure this result with 90 % confidence.

(8 × 5 = 40 marks)

Part B

2. (a) (i) Let V be the vector space of functions from $\mathbb{R}$ into $\mathbb{R}$. Show that $f, g, h \in V$ are linearly independent where $f(t) = e^{2t}$, $g(t) = t^2$, $h(t) = t$.

(8 marks)

Turn over

(ii) Let V be the vector space of polynomials $f(t)$ with inner product $\langle f, g \rangle = \int_{-1}^{1} f(t)\, g(t)\, dt$.

Apply the Gram - Schmidt algorithm to the set $\{1, t, t^2, t^3\}$ to obtain an orthonormal set $\{f_0, f_1, f_2, f_3\}$.

(7 marks)

Or

(b) (i) Consider the basis $B = \{u_1 = (1, 1, 0), u_2 = (1, 2, 3), u_3 = (1, 3, 5)\}$ of $\mathbb{R}^3$. Find the matrix A which represents the usual inner product on $\mathbb{R}^3$ with respect to the basis B.

(8 marks)

(ii) Find an orthonormal basis of the subspace w of $\mathbb{R}^5$ spanned by :

$v_1 = (1, 1, 1, 0, 1), v_2 = (1, 0, 0, -1, 1)$
$v_3 = (3, 1, 1, -2, 3), v_4 = (0, 2, 1, 1, -1)$.

(7 marks)

3. (a) (i) Find the Fourier transform of $f(x) = 1 - |x|$ if $|x| < 1$
$= 0$ for $|x| > 1$ and hence

find the value $\int_0^{\infty} \frac{\sin^4 t}{t^4}\, dt$.

(8 marks)

(ii) Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1 \text{ for } |x| < a \\ 0 \text{ for } |x| > a > 0 \end{cases}$ and hence evaluate

$\int_0^{\infty} \frac{\sin x}{x}\, dx$.

(7 marks)

Or

(b) (i) Express $f(x) = x(\pi - x)$, $0 < x < \pi$, as a Fourier series of periodicity 2π containing sine terms only.

(8 marks)

(ii) Find the Fourier series of periodicity 3 for $f(x) = 2x - x^2$ in $0 < x < 3$.

(7 marks)

4. (a) (i) Fit a normal curve to the following distribution :—

x	:	2	4	6	8	10
$f(x)$	:	1	4	6	4	1

(8 marks)

(ii) In a sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. out of 1,000 such samples, how many would be expected to contain atleast 3 defective parts.

(7 marks)

Or

(b) (i) If X and Y are independent random variable following N (8, 2) and N $(12, 4\sqrt{3})$ respectively. Find the value of λ such that

$$P(2X - Y \leq 2\lambda) = P(X + 2Y \geq \lambda).$$

(8 marks)

Or

(ii) A random variable X has a uniform distribution over (–3, 3), find k for which $P(X > k) = \frac{1}{3}$. Also, evaluate $P(X < 2)$ and $P(|X - 2| < 2)$.

(7 marks)

5. (a) (i) In two large populations there are 30 % and 25 % respectively of fair haired people. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations ?

(8 marks)

(ii) An unbiased coin is thrown n times. It is desired that the relative frequency of the appearance of heads should lie between 0.49 and 0.51. Find the smallest value of n that will ensure this result with 90 % confidence.

(7 marks)

Or

(b) Fit a normal distribution to the following data of wights of 100 students of Delhi university and test of the goodness of fit.

Weights (kg)	:	60.62	63.65	66.68	69.71	72.74
Frequency	:	5	18	42	27	8

(15 marks)

[4 × 15 = 60 marks]