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THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2010

EN 04 301 A—ENGINEERING MATHEMATICS—III

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

Part A

I. (a) Show that if 0 is one of the vectors $v_1, ..., v_m$, say $v_1 = 0$, then the vectors must be linearly dependent.

(b) Suppose $m > 1$, show that the vectors $v_1, ..., v_m$ are linear dependent if and only if one of them is a linear combination of the others.

(c) Find the fourier sine transform of $\frac{x}{a^2 + x^2}$.

(d) Expand the function $f(x) = \sin x$, $0 < x < \pi$ in Fourier cosine series.

(e) In 256 sets of 12 tosses of a coin, in how many cases one can expect 8 heads and 4 tails.

(f) A die is cast until 6 appears. What is the probability that it must be cast more than 5 times ?

(g) Give the 95% confidence interval of the population mean in terms of the mean and S.D. of a small sample.

(h) A sample of 900 members is found to have a mean of 3.4 cm. Can it be reasonably regarded as a truely random sample from a large population with mean 3.25 cm and S.D 1.61 cm.

(8 × 5 = 40 marks)

Part B

II. (a) (i) Let W be the subspace of $\mathbb{R}^4$ generated by the vectors :

$$(1, -2, 5, -3), (2, 3, 1, -4) \text{ and } (3, 8, -3, -5).$$

Find a basis and the dimension of W.

(8 marks)

(ii) Find an orthonormal basis of the subspace W of $\mathbb{R}^5$ spanned by $V_1 = (1, 1, 1, 0, 1)$, $V_2 = (1, 0, 0, -1, 1)$, $V_3 = (3, 1, 1 -2, 3)$, $V_4 = (0, 2, 1, 1, -1)$.

(7 marks)

Or

(b) (i) Explain how will you find an orthonormal basis from a given set of non-zero independent vectors.

(8 marks)

Turn over

(ii) Verify that the following is an inner product in $\mathbb{R}^2$:—

$$\langle u, v \rangle = x_1 y_1 - x_1 y_2 + 3x_2 y_2$$

where $u = (x_1, x_2), v = (y_1, y_2)$.

(7 marks)

III. (a) If $f(x) = \sin x$ in $0 \le x \le \pi$
$= 0$ in $\pi \le x \le 2\pi$

find a Fourier series of periodicity 2π and hence evaluate :

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots \text{ to } \infty$$

(15 marks)

Or

(b) Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1 \text{ for } |x| < a \\ 0 \text{ for } |x| > a > 0 \end{cases}$ and hence evaluate

$$\int_0^\infty \frac{\sin x}{x} dx.$$

(15 marks)

IV. (a) (i) In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours. Estimate the no. of bulbs likely to burn for (a) more than 2150 hours ; (b) less than 1950 hours ; and (c) more than 1920 hours and but less than 2160 hours.

(8 marks)

(ii) In sampling a large no . of parts manufactured by a machine, the mean no. of defectives in a sample of 20 in 2. Out of 1000 such samples, how many would be expected to contain at least 3 defective parts.

(7 marks)

Or

(b) (i) A random variable X has a uniform distribution over $(-3, 3)$, find k

$P(X > k) = \frac{1}{3}$. Also evaluate $P(X < 2)$ and $P\{|X - 2| < 2\}$.

(8 marks)

(ii) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. of the distribution.

(7 marks)

V. (a) (i) Balls are drawn from a bag containing equal number of black and white balls, each ball being replaced before drawing another. In 2250 drawings 1018 black and 1232 white balls have been drawn. Do you suspect some bias on the part of the drawer ?

(8 marks)

(ii) In a random sample of size 500, the mean is found to be 20. In another independent sample of size 400, the mean is 15. Could the samples have been drawn from the same population with S.D. 4 ?

(7 marks)

Or

(b) A set of five similar coins is tossed 320 times and the result is :

No. of heads	:	0	1	2	3	4	5
Frequency	:	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution.

(15 marks)

[4 × 15 = 60 marks]