D 51010 **(Pages : 3)** Name.................................

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THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION NOVEMBER 2013

Mathematics

EN 09 301—ENGINEERING MATHEMATICS—III

Time : Three Hours Maximum : 70 Marks

Part A

Answer **all** *questions.*

1. Define Harmonic functions.
2. Find the image of the circle $|z| = a$ under the transformation $w = (3+4i)\, z$.
3. State Cauchy's integral formula.
4. Show that the intersection of any two subspaces U and W of a vector space V is also a subspace.
5. Find the Fourier Cosine transform of $e^{-ax} (a > 0)$.

(5 × 2 = 10 marks)

Part B

Answer any **four** *questions.*

6. Find the analytic function $w = u + iv$, if $v = e^{-x}(x \cos y + y \sin y)$. Hence find u.
7. Find the image of the semi-infinite strip $0 \le x \le \pi, y \ge 0$ under the transformation $w = \cos z$.
8. Find the residues at the isolated singularities of the function $\dfrac{z^2}{z^2 + a^2}$.
9. Determine whether the vectors (2, 1, 1), (5, 2, 1) and (4, 3, 5) form a basis for the vector space R^3.
10. Find the Fourier transform of $f(x)$ if

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$$

Hence deduce that

Turn over

(a) $\int_0^\infty \frac{\sin t}{t}\, dt = \frac{\pi}{2}$.

(b) $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$.

11. Show that any set of *three* vectors in $\mathbb{R}^2$ is linearly dependent.

(4 × 5 = 20 marks)

Part C

Answer **all** *questions as per choice given.*

12. (a) If $f(z) = \sqrt{|xy|}$, prove that $f(z)$ satisfies Cauchy–Riemann equations at the origin but it is not analytic at the origin.

Or

(b) Prove that $w = \frac{z}{1-z}$ maps the upper half of the z–plane onto the upper half of the w–plane. What is the image of the circle $|z| = 1$ under this transformation ?

13. (a) Obtain Laurent's expansion for $f(z) = \frac{1}{(z-1)(z-2)}$ valid in the regions :

(i) $|z-1| < 1$.

(ii) $|z| > 2$.

Or

(b) Using Contour integration show that

$$\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta}\, d\theta = \frac{\pi}{12}.$$

14. (a) Let U and W be the subspaces of $\mathbb{R}^4$ generated by {(1, 1, 0, –1), (1, 2, 3, 0), (2, 3, 3, –1)}, and {(1, 2, 2, –2), (2, 3, 2, –3), (1, 3, 4, –3)} respectively, find

(i) dim (U + W).

(ii) dim (U ∩ W).

Or

(b) Verify that the vectors $\left(\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}\right); \left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right)$ and $\left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right)$ form an orthonormal basis for $V_3(R)$ relative to the standard inner product.

15. (a) If $F[f(x)] = \bar{f}(s)$, then show that $F[f(ax)] = \frac{1}{|a|}\bar{f}\left(\frac{s}{a}\right)$.

Or

(b) Find the Fourier sine transform of $e^{-ax}(a > 0)$. Hence find $F_s\{xe^{-ax}\}$ and $F_s\left\{\frac{e^{-ax}}{x}\right\}$.

(4 × 10 = 40 marks)