C 41281 (Pages : 4) Name..............................

Reg. No..............................

SIXTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, MAY 2013

ME/PTME 09 604—OPERATION RESEARCH

(2009 Admission onwards)

Time : Three Hours Maximum : 70 Marks

Part A

Answer **all** *questions.*

1. Define Operation Research.
2. What is meant by feasible solution of a linear programming model.
3. What is an unbounded solution ?
4. What is degeneracy in transportation problem ?
5. State Bellman's principle of optimality.

(5 × 2 = 10 marks)

Part B

Answer any **four** *questions.*

6. A firm manufactures two types of products, A and B and sells them at a profit of Rs. 2/- on type A and Rs. 3/- on type B. Each product is processed on two machine M and N. Type A requires one minute of processing time on M and two minutes on N; type B requires one minute on M and one minute on N. The machine M is available for not more than 6 hours 40 minute while machine N is available for 10 hours during any working day. Formulate the problem as a linear programming problem.
7. Express the following LPP in the standard form :

Maximise $Z = 2x_1 + 3x_2 + x_3$

subjected to constraints

$$4x_1 - 3x_2 + x_3 \le 6$$

$$x_1 + 5x_2 - 7x_3 \ge -4$$

$$x_1, x_3 \ge 0, \; x_2 \text{ is unrestricted}$$

Turn over

8. Determine the dual of the problem :

$$\begin{aligned} \text{Minimise } Z &= 5x_1 + 2x_2 + x_3 \\ \text{subject to } \quad & 2x_1 + 3x_2 + x_3 \geq 20 \\ & 6x_1 + 8x_2 + 5x_3 \geq 30 \\ & 7x_1 + x_2 + 3x_3 \geq 40 \\ & x_1 + 2x_2 + 4x_3 \geq 50 \\ & \text{and } x_1, x_2, x_3 \geq 0. \end{aligned}$$

9. Four new machines M_1, M_2, M_3, M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D, E that are available. Because of limited space, machine M_2 cannot be placed at C and M_3 cannot placed at A. The cost matrix is shown below :

	A	B	C	D	E
M_1	4	6	10	5	4
M_2	7	4	–	5	4
M_3	–	6	9	6	2
M_4	9	3	7	2	2

Find the optimal assignment schedule.

10. For what values of λ, the game with the following matrix is determinable :

$$\begin{array}{cc} & \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} \lambda & 6 & 2 \\ -1 & \lambda & -7 \\ -2 & 4 & \lambda \end{bmatrix} \end{array}$$

Treating λ is neither minimax nor Maximin

11. At what average must a clerk at a supermarket work in order to ensure a probability of 0.90 that the customer will not have to wait longer than 12 minutes ? It is assumed that there is only one counter to which customers arrive in a Poission fashion at an average rate of 15 per hour. The length of service by the clerk has an exponential distribution.

(4 × 5 = 20 marks)

Part C

12. (a) Prove that the collection of all feasible solution to linear programming problems constitute a convex set whose extreme points correspond to the basic feasible solutions.

Or

(b) Solve using graphical method :

$$
\begin{aligned}
\text{Maximise } Z &= x_1 - 2x_2 \\
\text{subjected to, } -x_1 + x_2 &\leq 1 \\
6x_1 + 4x_2 &\geq 24 \\
0 \leq x_1 &\leq 5, \\
2 \leq x_2 &\leq 4.
\end{aligned}
$$

13. (a) Prove that if an LPP has a feasible solution, then it also has a basic feasible solution.

Or

(b) Use simplex method solve the LPP :

$$
\begin{aligned}
\text{Maximise } Z &= 7x_1 + 9x_2 \\
\text{subjected to } -x_1 + 3x_2 &\leq 6 \\
7x_1 + x_2 &\leq 35 \\
x_1, x_2 &\geq 0 \text{ and are integers.}
\end{aligned}
$$

14. (a) A company has three plants at locations A, B, C which supply to warehouse located at D, E, F, G and H. Monthly plant capacities are 800, 500 and 900 units, respectively. Monthly warehouse requirement are 400, 400, 500, 400 and 800 units, respectively unit transportation cost (in Rs.) are given below. Determine an optimum distribution for the company in order to minimise the total transportation cost.

To

		D	E	F	G	H
	A	5	8	6	6	3
From	B	4	7	7	6	5
	C	8	4	3	6	4

Or

(b) Solve the assignment problem represented by the matrix.

	1	2	3	4	5	6
A	9	22	58	11	19	27
B	43	78	72	50	63	48
C	41	28	91	37	45	33
D	74	42	27	49	39	32
E	36	11	57	22	25	18
F	3	56	53	31	17	28

15. (a) Obtain the optimal strategies for both pearsons and the value of the game for zero-sum two-person game whose pay-off matrix is as follows :

$$\text{Player A}\quad \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{array}\!\!\begin{array}{c} \text{Player B} \\ \begin{array}{cc} B_1 & B_2 \end{array} \\ \left[\begin{array}{cc} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{array}\right]. \end{array}$$

Or

(b) A vessel is to be loaded with stocks of three items. Each unit of item i has a weight w_i and value r_i. The maximum cargo weight the vessel can take is 5 and the details are as follows :

i	w_i	r_i
1	1	30
2	3	80
3	2	65

Find the most valuable cargo load without exceeding the maximum cargo weight by using dynamic programming.

(4 × 10 = 40 marks)