C 18260 (Pages : 2) Name..............................

Reg. No..............................

SIXTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2011

CS 04 604—GRAPH THEORY AND COMBINATORIES

(2004 admissions)

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

I. (a) Define planar graphs and show that complete graph on five vertices is not planar.

(b) Explain how to compute chromatic number of a graph G, if chromatic polynomial is given.

(c) Write Prim's algorithm.

(d) Write the condition on the tree having a maximum matching covering all the vertices.

(e) Prove that $C(n, 0) + C(n, 1) + \cdots + C(n, n) = 2^n$.

(f) Compute $\phi(n)$ for n equal to (i) 51 ; (ii) 420.

(g) Solve $a_{n+2} - 3a_{n+1} + 2a_n = 0, a_0 = 2, a_1 = 3$.

(h) Calculate $B(x) = \sum_{r=0}^{\infty} b_r x^r = \frac{1}{x^2 - 5x + 6}$.

(8 × 5 = 40 marks)

II. (a) State and prove five colour theorem for planar graphs.

Or

(b) Show that every graph with minimum degree at least $p/2$, where p is number of vertices, has Hamilton cycle and hence no cut vertices.

(15 marks)

III. (a) Explain Bellnan-Ford algorithm with an example.

Or

(b) State and prove the max-flow min-cut theorem.

(15 marks)

IV. (a) (i) Prove any three properties of binomial coefficients. (8 marks)

(ii) Find the non-negative integer solutions to the equation $x_1 + x_2 + x_3 + x_4 = 18$, $x_i \le 7$, for all $1 \le i \le 4$.

(7 marks)

Or

(b) In how many ways can we distribute eight identies white balls into four distinct containers so that (i) no container is left empty (ii) the fourth container has an odd number of balls in it.

V. (a) (i) Find the generating function of $\sum_{n=0}^{\infty} n^3 a^n x^n$. (7 marks)

(ii) Find the particular solution to $a_n - 10a_{n-1} + 25a_{n-2} = 2^n$ where $a_0 = 2/3$ and $a_1 = 3$. (8 marks)

Or

(b) (i) Solve $a_n - 6a_{n-1} + 8a_{n-2} = n4^n$ where $a_0 = 8$ and $a_1 = 22$. (9 marks)

(ii) By the method of generating function solve $a_{n+1} - a_n = n^2, n \geq 0, a_0 = 1$. (6 marks)

[4 × 15 = 60 marks]