**D 34510** (Pages : 3)

Name.............................................

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# FOURTH SEMESTER B.TECH. (ENGINEERING) DEGREE [2K SCHEME] EXAMINATION, MARCH 2013

AI/CH/EE/EC/IC/ME/PE 2K 401/CE 2K 401—ENGINEERING MATHEMATICS—IV

Time : Three Hours Maximum : 100 Marks

*Answer* **all** *questions.*

I. 1 If $w = u(x,y) + iv(x,y)$ is an analytic function of $z$ in a region R, then in R prove that the curves of the family $u(x,y) = c$ are orthogonal trajectories of the curves of the family $v(x,y) = \mathrm{R}$.

2 What is the bilinear transformations which sends the points $z = 0, i, -i$ into the points $w = i, i, i/2$ respectively.

3 Evaluate $\int_C \frac{\sin^2 z\, dz}{\left(z - \pi/6\right)^3}$, where C is the circle $|z| = 2$.

4 Find the Taylor expansion of $f(z) = \frac{z+3}{5z = z^2}$ around $z = 1$.

5 For all values of $\gamma, \frac{d}{dx}\left(x^{-\gamma} J_\gamma(x)\right) = -x^{-\gamma} J_{\gamma+1}(x)$.

6 Using power series method, solve $\frac{d^2y}{dx^2} + y = 0$.

7 Find the characteristics of the pde $xu_{xy} + yu_{yy} = 0$.

8 Show that $f(x) = 1$ and $g(x) = x$ are orthogonal on $(-1,1)$ and find constants $a$ and $b$ such that $h(x) = 1 + ax + bx^2$ will be orthogonal to both $f(x)$ and $g(x)$ on $(-1,1)$.

(8 × 5 = 40 marks)

II. A (a) Show that $u = e^x \sin y$ is a harmonic function and determine $v$ so that $u + iv$ is an analytic function.

(b) If $u$ and $v$ are harmonic in a region R, show that $\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ is analytic in R.

*Or*

B (a) What is the most general bilinear transformation which maps the upper half of the $z$-plane onto the interior of the unit circle in the $w$-plane.

(b) Discuss the way in which the $z$-plane is mapped onto the $w$-plane by the function $w = z^2$.

(15 marks)

III. A (a) Evaluate $\int_C \frac{1}{z^2(z^2+6z+4)} dz$ where C is the circle $|z| = 4$.

(b) Find the Laurent series expansion of the function $f(x) = \frac{5z+7}{(z+1)(z)(z-3)}$ in the annulus $1 < |z+1| < 3$.

*Or*

B (a) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$ where $a$, $b$ positive.

(b) Evaluate $\int_0^{2\pi} \frac{\cos\theta \, d\theta}{5+4\cos\theta}$.

(15 marks)

IV. A (a) Prove that $\sum_{n=0}^{\infty} \mathcal{P}_n(x) z^n = (1-2xz+z^2)^{-1/2}$.

(b) Express the polynomial $x^4 + 5x^3 + 3x^2 + 2x + 7$ in terms of Legendre's polynomials.

*Or*

B (a) Solve $x^2y'' - xy' + (1+x)y = 0$ in terms of Bessel's functions.

(b) Show that $\int J_1(x)\, dx = -J_0(x) + c$.

(15 marks)

V. A (a) Solve the one dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ by the method of separation of variables.

(b) Find a particular solution of the equation $\frac{\partial^2 u}{\partial x^2} = -\frac{\partial u}{\partial y} = 2e^{2x+3y}$.

*Or*

B (a) A uniform string stretched from $x = 0$ to $x = \pi$ is given the initial displacement $y(x, 0) = \sin x$ and released from rest in that position. Find the subsequent displacement of the string as a function of $x$ and $t$.

(b) A semi infinite string is initially at rest in a position coinciding with the positive half of the $x$-axis. At $t = 0$ the left hand end of the string begins to move along the $y$-axis in a manner described by $y(0,t) = f(t)$ where $f(t)$ is a known function. Find the displacement $y(x,t)$ of the string at any point at any subsequent time.

(15 marks)

[4 × 15 = 60 marks]