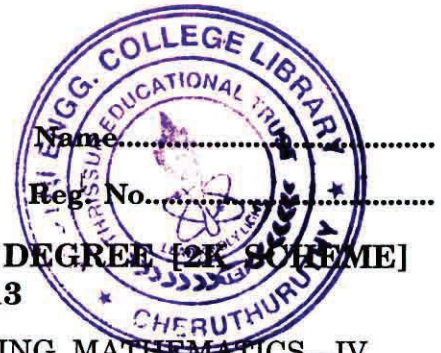


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**FOURTH SEMESTER B.TECH. (ENGINEERING) DEGREE [2K SCHEME]
EXAMINATION, MARCH 2013**

AI/CH/EE/EC/IC/ME/PE 2K 401/CE 2K 401—ENGINEERING MATHEMATICS—IV

Time : Three Hours

Maximum : 100 Marks

Answer all questions.

- I. 1 If $w = u(x, y) + iv(x, y)$ is an analytic function of z in a region R , then in R prove that the curves of the family $u(x, y) = c$ are orthogonal trajectories of the curves of the family $v(x, y) = R$.
- 2 What is the bilinear transformations which sends the points $z = 0, i, -i$ into the points $w = i, i, i/2$ respectively.
- 3 Evaluate $\int_C \frac{\sin^2 z dz}{(z - \pi/6)^3}$, where C is the circle $|z| = 2$.
- 4 Find the Taylor expansion of $f(z) = \frac{z+3}{5z-z^2}$ around $z = 1$.
- 5 For all values of γ , $\frac{d}{dx} (x^{-\gamma} J_{\gamma}(x)) = -x^{-\gamma} J_{\gamma+1}(x)$.
- 6 Using power series method, solve $\frac{d^2 y}{dx^2} + y = 0$.
- 7 Find the characteristics of the pde $xu_{xy} + yu_{yy} = 0$.
- 8 Show that $f(x) = 1$ and $g(x) = x$ are orthogonal on $(-1, 1)$ and find constants a and b such that $h(x) = 1 + ax + bx^2$ will be orthogonal to both $f(x)$ and $g(x)$ on $(-1, 1)$.

(8 × 5 = 40 marks)

II. A (a) Show that $u = e^x \sin y$ is a harmonic function and determine v so that $u + iv$ is an analytic function.

(b) If u and v are harmonic in a region R , show that $\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ is analytic in R .

Or

Turn over

B (a) What is the most general bilinear transformation which maps the upper half of the z -plane onto the interior of the unit circle in the w -plane.

(b) Discuss the way in which the z -plane is mapped onto the w -plane by the function $w = z^2$.
(15 marks)

III. A (a) Evaluate $\int_C \frac{1}{z^2(z^2 + 6z + 4)} dz$ where C is the circle $|z| = 4$.

(b) Find the Laurent series expansion of the function $f(z) = \frac{5z + 7}{(z + 1)(z)(z - 3)}$ in the annulus $1 < |z + 1| < 3$.

Or

B (a) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$ where a, b positive.

(b) Evaluate $\int_0^{2\pi} \frac{\cos \theta d\theta}{5 + 4 \cos \theta}$.

(15 marks)

IV. A (a) Prove that $\sum_{n=0}^{\infty} P_n(x) z^n = (1 - 2xz + z^2)^{-1/2}$.

(b) Express the polynomial $x^4 + 5x^3 + 3x^2 + 2x + 7$ in terms of Legendre's polynomials.

Or

B (a) Solve $x^2 y'' - xy' + (1 + x)y = 0$ in terms of Bessel's functions.

(b) Show that $\int J_1(x) dx = -J_0(x) + c$.

(15 marks)

V. A (a) Solve the one dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ by the method of separation of variables.

(b) Find a particular solution of the equation $\frac{\partial^2 u}{\partial x^2} = -\frac{\partial u}{\partial y} = 2e^{2x+3y}$.

Or

- B (a) A uniform string stretched from $x = 0$ to $x = \pi$ is given the initial displacement $y(x, 0) = \sin x$ and released from rest in that position. Find the subsequent displacement of the string as a function of x and t .
- (b) A semi infinite string is initially at rest in a position coinciding with the positive half of the x -axis. At $t = 0$ the left hand end of the string begins to move along the y -axis in a manner described by $y(0, t) = f(t)$ where $f(t)$ is a known function. Find the displacement $y(x, t)$ of the string at any point at any subsequent time.

(15 marks)

[4 × 15 = 60 marks]