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Name : ....

Reg. No : ....

COMBINED FIRST AND SECOND SEMESTER B.TECH (ENGINEERING) DEGREE EXAMINATION, APRIL 2013

EN 2K 102 – MATHEMATICS – II
(Common to All Branches)

Time : Three Hours **Maximum : 100 Marks**

Answer all questions.

Part A

I (a) Solve $\left(\cos x \tan y + \cos(x+y)\right)dx + \left[\sin x \sec^2 y + \cos(x+y)\right]dy = 0$.

(b) Solve $\left(D^3 - 6D^2 + 11D - 6\right)y = e^{-3x}$.

(c) Prove that $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$.

(d) Evaluate $\lfloor te^{-t}\sin^2 t$.

(e) Prove that $\nabla.\left(\phi\overline{A}\right) = \phi\nabla.\overline{A} + \nabla\phi.\overline{A}$.

(f) Find the directional derivative of the function $f = x^2 - y^2 + 2z^3$ at the point $p(1,2,3)$ in the direction of the line PQ where Q is the point (5, 0, 4).

(g) If $\overline{F} = \left(3x^2 + 6y\right)i - 14yzj + 20xz^2k$ evaluate the line integral $\int_c \overline{F}.d\overline{r}$ from $(0,0,0)$ to $(1,1,1)$ along the path c represented by $x = t,\ y = t^2,\ z = t$.

(h) Find $\iint_s \overline{F}.\hat{n}\, ds$ where $\overline{F} = (2x+3z)i - (xz+y)j + \left(y^2 + 2z\right)k$ and s is the surface of the sphere having centre at $(3,-1,2)$ and radius 3.

(8 x 5 = 40 Marks)

Part B

II (a) (i) Solve $\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = xe^x \sin x$. (7)

(ii) Solve by method of variation of parameters, the differential equation $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$. (8)

(Or)

(b) (i) Find the orthogonal trajectories of the family of ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (7)

(ii) An electric circuit consists of an inductance of 0.1 henry, a resistance of 20 ohms and a condenser of capacitance 25 micro-farads. Find the charge q and the current i at any time t, given that at $t = 0$, $q = 0.05$ coulomb, $i = \frac{dq}{dt} = 0$ when $t = 0$. (8)

III (a) (i) Find $L\left(\int_0^t e^t \frac{\sin t}{t} dt\right)$ (7)

(ii) Find $L^{-1}\left(\log\left(\frac{s(s+1)}{s^2+4}\right)\right)$ (8)

(Or)

Turn over

(b) (i) Find the Laplace transform of the rectified semi-wave function defined by

$$f(t) = \begin{cases} \sin wt, & 0 < t < \dfrac{\pi}{w} \\ 0, & \dfrac{\pi}{w} < t < \dfrac{2\pi}{w} \end{cases}.$$ (7)

(ii) Solve $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 3y = e^{-x}$, $y(0) = y'(0) = 1$. (8)

IV (a) (i) Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point $(1,-2,1)$. (7)

(ii) Find the constants a, b, c so that $\overline{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is irrotational. Find a scalar function ϕ such that $\overline{F} = grad\,\phi$. (8)

(Or)

(b) (i) If $\overline{r} = xi + yj + zk$ prove that

(1) $div\left(r^n \overline{r}\right) = (n+3)r^n$.

(2) $curl\left(r^n \overline{r}\right) = \overline{0}$.

(3) $\dfrac{\overline{r}}{r^3}$ is solenoidal. (7)

(ii) If $\overline{F} = grad\left(x^3 + y^3 + z^3 - 3xyz\right)$ find div $\overline{F}$, curl $\overline{F}$, curl curl $\overline{F}$, div curl $\overline{F}$. (8)

V (a) Verify Green's theorem in the plane for $\oint_c \left(xy + y^2\right)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. (15)

(Or)

(b) Verify divergence theorem for $\overline{F} = yi + xj + z^2k$ over the cylindrical region bounded by $x^2 + y^2 = a^2$, $z = 0$ and $z = h$. (15)

(4 x 15 = 60 Marks)

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